Nonlinear Excitation of Zonal Flow and Geodesic Acoustic Modes in the Edge of Tokamak Plasma

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A self-consistent model of the multiscale interaction of zonal flows (ZFs), zonal fields (ZFLDs) and geodesic acoustic mode (GAMs) with edge turbulence is presented. In the collisionless and collisional regimes, the dominant short scale modes in the background at the edge of tokamak plasmas are electrostatic ion temperature gradient (ITG) mode and electromagnetic high-m drift-resistive-ballooning mode (DRBM), respectively. The modulational instability of ZFs, ZFLDs, and GAMs in the background of ITG and DRBM driven turbulence has been studied. The kinetic wave equation is used to study the adiabatic interaction between long-scale ZFs, ZFLDs and GAMs, and the small-scale background turbulence (DRBM, ITG). We have also determined conditions under which ZFs saturate by different mechanics such as by (i) collisional damping (ii) instability to tertiary modes (iii) nonlinear trapping of ITG mode turbulence in ZFs, giving coherent structures etc. From a 'predator-prey' model, the turbulent transport in a collisional edge (Low-confinement regime) has been estimated. We have also derived the condition for density limit from the threshold of the tertiary modes. The dependence of critical density n_{cr} on other basic plasma parameters is also given.

INTRODUCTION: The physics of radial turbulent transport in tokamak edge plasma is a topic of great current interest for optimizing the performance of ITER and other tokamaks. It is now widely accepted that the edge physics plays a vital role in controlling the global confinement properties of a tokamak discharge through phenomena such as the L–H transition, Greenwald density limit disruption etc. Furthermore, an agreement seems to be emerging around the view that when the plasma is collisional, ($C = \lambda_f / qR$; λ_f, the collisional mean free path, q, the safety factor, R, the major radius), the edge plasma collisionality (C) plays a crucial role in determining the underlying physical processes. For example, if $C < 1$, the turbulence process in the edge maybe dominated by the nonlinear properties of high-m drift resistive ballooning mode (DRBM) [1-2], which has a typical growth rate of order the ideal growth rate, $\gamma_{ideal} \sim c_s\sqrt{2/RL_n}$, $c_s = \sqrt{T_e/m_i}$, the ion sound speed, and L_n, the density gradient scale length. On the other hand, if $C > 1$, which can be expected to occur in ITER-like machines, especially with sharp gradients of ``equilibrium'' profiles, the edge plasma may well be in a nearly collisionless regime. In such situations, the turbulence may be dominated by properties of the ITG modes. An intermediate regime of $C \simeq 1$, is by far a complex situation where weak DRBM, weak ITG and/or drift-Alfven modes could become important. In this paper we investigate the nonlinear excitation of ZFs[3], ZFLDs, and GAMs[4-6] in the two distinct regimes of collisionality and explore the modulational stability of the primary high-m DRBM/ITG turbulence to the long-scale modes. Since the long-scale ZF, ZFLD, and GAM modes are well separated from the small-scale modes driving them (i.e., ITG and high-m DRBM turbulence), the wave-kinetic-equation and adiabatic theory is used to study the interaction between these modes.

The predator-prey model for saturation of ZFs shows that the saturation level of the primary turbulence is determined by the damping of the secondary ZF modes. In the collisional limit (C < 1), damping of ZFs is dominated by neoclassical collisional damping and we get a primary turbulence saturation level, proportional to neoclassical collisional damping. As collisional damping decreases, the saturation level drops, anomalous transport diminishes and we go from L to H modes. When collisional damping is very small, the model predicts unrealistically low values of the saturated primary turbulence and transport. Under these conditions we must look for other mechanisms, which take the place of collisional damping of ZFs. We consider two mechanisms in this paper, which are particularly relevant for the regime where the neoclassical damping is smaller than ZFs growth. First is a tertiary instability mechanism in which the secondary zonal flows and fields are themselves unstable to a Kelvin-Helmholtz (K-H) type of instability. We find that the ZFLDs provide an additional effective magnetic shear and actually stabilize the tertiary instability. It is important to note that in a time-dependent situation, short-scale and long-scale ZFs, ZFLDs, and tertiary modes could co-exist (predator-prey) and hunt for the source of free energy. Then, a quasi-steady state of such a system could only be determined by the comparison of their relative growth rates to the neoclassical collisional damping rate. Thus for example, (1) an L-mode could be thought of as a state where the growth of ZFs is less or comparable to ion neoclassical collisional rate (i.e. $\nu_i^{neo} \geq \gamma_{ZF}$), (2) an H- mode can be attained when the ZFs growth rate is larger than neoclassical collisional damping, as well as the ZF is stable to the tertiary wave (K-H instability) i.e., $\gamma_{K-H} < \nu_i^{neo} < \gamma_{ZF}$, (3) the Greenwald density limit could be hit if the ZF is unstable to the tertiary mode i.e., the growth rate of tertiary wave $\gamma_{K-H} \geq 0$. Making use of the condition $\gamma_{K-H} \approx 0$, we get the dependence of critical density n_{cr} on other plasma parameters: $n_{cr} \sim I^{0.93} R^{0.58} B^{0.81} / A^{0.12}$. Another mechanism which operates in some collisionless cases, where dispersion is important, is the trapping effect of primary drift-like modes in secondary zonal flows, which produce nonlinear coherent structures with saturated drift-like turbulence and ZFs nonlinearly sustaining each other with trapped and untrapped distributions of primary turbulence. We illustrate this mechanism with an example of collisionless ITG modes trapped in ZFs. This required condition for trapping is $\nu_i^{neo} < \gamma_{K-H} < \gamma_{ZF} < \omega_B$, where ω_B is the bounce frequency of trapped particle in the zonal potential field.

Model equations: In the limit $k_{//} c_e > \omega >> k_{//} c_s$, the basic equations describing the dynamics of DRBM/ITG are the equations for perturbations of density $\tilde{n}$, potential $\tilde{\phi}$, temperature $\tilde{T}_i$, magnetic flux $\tilde{A}_{\|}$

$$\begin{aligned}\partial_t \tilde{n}_k + (1-\varepsilon_n)\partial_y \tilde{\phi}_k - \tau_i \varepsilon_n \partial_y (\tilde{n}_k + \tilde{T}_{ik}) - (\partial_t - \alpha_i \partial_y)\nabla_\perp{}^2 \tilde{\phi}_k \\ = -[\tilde{\phi}_k, \tilde{n}_k] + [\tilde{\phi}_k, \nabla_\perp{}^2 \tilde{\phi}_k] + \tau_i \vec{\nabla} \cdot [\hat{\phi}_k, \vec{\nabla}(\tilde{n}_k + \tilde{T}_{ik})]\end{aligned} \quad (1)$$

$$(\partial_t - \alpha_i \partial_y)\nabla_\perp{}^2 \tilde{\phi}_k + \varepsilon_n \partial_y [(1+\tau_i)\tilde{n} + \tau_i \tilde{T}_{ik}] + \nabla_{//}\nabla_\perp{}^2 \tilde{A}_{//k} = -[\tilde{\phi}_k, \nabla_\perp{}^2 \tilde{\phi}_k] + 0.5\beta[\tilde{A}_{//k}, \nabla_\perp{}^2 \tilde{A}_{//k}] \quad (2)$$

$$\partial_t \tilde{T}_{ik} + (\eta_i - 2/3)\partial_y \tilde{\phi}_k - (5\tau_i/3)\varepsilon_n \partial_y \tilde{T}_{ik} - (2/3)\partial_t \tilde{n}_k = -[\tilde{\phi}_k, \tilde{T}_{ik}] \quad (3)$$

$$-\nabla_\perp{}^2 \tilde{A}_{//k} + 0.5\beta\hat{\chi}_e(\partial_t + \alpha_e \partial_y)\tilde{A}_{//k} + \hat{\chi}_e \nabla_{//}(\tilde{\phi}_k - \tilde{n}_k) = -0.5\beta\hat{\chi}_e[(\tilde{\phi}_k - \tilde{n}_k), \tilde{A}_{//k}] \quad (4)$$

Here note that for simplicity we have neglected ion and electron temperature perturbations in the nonlinear terms, however, finite-Larmor-radius effect through polarization drift due to diamagnetic effect is included. The various plasma parameters and perturbed quantities are defined as in Ref. (1).

Linear dispersion relation of High –m DRBM: The linear dispersion relation for the high-m Drift Resistive Ballooning mode (DRBM) [2] is:

$$\varepsilon_0(\omega, \vec{k}) = k_\perp^2 \omega(\omega + \alpha_i k_y - \varepsilon_n k_y) + \varepsilon_n k_y^2[(1-\varepsilon_n)(1+\tau_i) - \alpha_i k_\perp^2] + i\hat{\chi}_e k_{//}^2[\omega - k_y + \varepsilon_n k_y + k_\perp^2(\omega + \alpha_i k_y)][1 - i0.5\hat{\beta}\hat{\chi}_e(\omega - k_y)/k_\perp^2]^{-1} = 0 \quad (5)$$

This equation contains a simple drift mode, a standard DRBM and also the high-m DRBM (or new branch of DRBM) [1]. In the limit $k_\perp^2 \omega < \hat{\chi}_e k_\parallel^2$, $0.5\hat{\beta}\hat{\chi}_e(\omega - k_y) < k_\perp^2$ and without ion temperature perturbation ($\tilde{T}_{ik} = 0$), the frequency and growth rate of electrostatic drift and drift interchange modes are

$$\omega_{drift} \approx k_y(1 - \alpha_i k_\perp^2)\big/(1 + k_\perp^2), \quad \gamma_{d/\text{int}erchang} \approx \left[k_\perp^2 \omega_{drit}(\omega_{drit} + \alpha_i k_y) + \varepsilon_n k_y^2(1+\tau_i)\right]\big/\hat{\chi}_e k_{//}^2 \quad (6)$$

The DRBM instability dominates over drift-Alfven mode, simple drift, and resistive ballooning mode if the plasma is collisional,

$k_\perp^2 > [\sqrt{2}\ \lambda_f / q^2 R\sqrt{1+\tau_i}](L_n m_i / R m_e)^{1/2} > \sqrt{2(1+\tau_i)}\ \beta(\lambda_f / R)(R m_i / L_n m_e)^{1/2}$.

This also yields the condition that β is less than the ideal β_c [i.e. $\beta_c \approx (1+\tau_i)\beta q^2 R / L_n) < 1$]. In the limit $\varepsilon_n < 1$ and $k_\perp^2 < 1$, the real frequency and growth rate of DRBM are given by

$$\omega_r \approx -0.5\alpha_i k_y + \omega_1(k_\parallel); \quad \omega_1(k_\parallel) \approx 0.25 k_y[2 + \alpha_i(1 - k_\perp^2)]\hat{\chi}_e k_\parallel^2 / k_\perp^2 \gamma_0$$
$$\gamma \approx \gamma_0 + \gamma_1(k_\parallel); \quad \gamma_0 = [\varepsilon_n(1+\tau_i) - \alpha_i^2 k_y^2 / 4]^{1/2}, \quad \gamma_1(k_\parallel) \approx -0.5(1 + k_\perp^2)\hat{\chi}_e k_\parallel^2 / k_\perp^2 \quad (7)$$

Here γ_0 is the ideal mode growth rate. Note that $k_\parallel$ and electric conductivity ($\hat{\chi}$) effects are stabilizing and also introduce a frequency shift in the electron diamagnetic direction. One can also note that the growth rate of DRBM is close to the ideal growth rate when the plasma is highly collisional.

Basic equation for long scale modes in background of high – m DRBM wave turbulence: The standard description for generation of ZFs, ZFLDs, GAMs, and tertiary waves in the background of short-scale turbulence (DRBM) relies on two-scale separation. The ZFs, ZFLDs, and GAMs describing large-scale waves, vary on a longer time scale compared to the small-scale DRBM fluctuations - there is a sufficient spectral gap separating both the scales (i.e. $q_\perp < k_\perp$; $q_\perp$ and $k_\perp$ are the perpendicular wavelength associated with ZFs, ZFLDs, GAMs, and DRBM modes, respectively). We therefore exploit the two-scale assumption to obtain an equation for the long-scale modes by averaging over fast times and shorter structures. In the limit

$q_{\perp}^2 < 2\beta(1+\tau_i)(\lambda_f / R)(R/L_n)^{1/2}(m_i/m_e)^{1/2}$ and $L_n < L_T$, the basic equations for the long wavelength modes are:

$$(\partial_t - \alpha_i \partial_y + \nu_i^{neo})\nabla_\perp{}^2\tilde{\phi}_q + (1+\tau_i)\varepsilon_n \partial_y \tilde{n}_q + \nabla_{//}\nabla_\perp{}^2 \tilde{A}_{//q} = -\left\langle [\tilde{\phi}_k, \nabla_\perp{}^2 \tilde{\phi}_k] \right\rangle + 0.5\beta \left\langle [\tilde{A}_{//k}, \nabla_\perp{}^2 \tilde{A}_{//k}] \right\rangle \quad (8)$$

$$(\partial_t + \alpha_e \partial_y)\tilde{A}_{//q} + 2\beta^{-1}\nabla_{//}(\tilde{\phi}_q - \tilde{n}_q) = -[(\tilde{\phi}_k - \tilde{n}_k), \tilde{A}_{//k}] \quad (9)$$

Here, the average $\langle\, [] \,\rangle$ indicates an averaging over small scales.

The effect of high-frequency DRBM fluctuations with random phases on long-scale ZFs, ZFLDs, and GAMs can be estimated from the Wave-Kinetic Equation (WKE)

$$\partial_t N_k + \partial_{\vec{k}}\omega_{nl} \cdot \partial_{\vec{x}} N_k - \partial_{\vec{x}}\omega_{nl} \cdot \partial_{\vec{k}} N_k = \gamma_{nl} N_k - \Delta\omega_k N_k^2 \ . \quad (10)$$

The quantity N_k is the adiabatic action invariant [3,8], where $N_k = E_k / |\omega_{rk}|$ and E_k is the energy of mode k with frequency $|\omega_{rk}|$. In Fourier space, E_k of each mode k is $E_k = |\tilde{n}_k|^2 + k_\perp^2 |\tilde{\phi}|^2 + |\tilde{A}_{\parallel k}|^2 \approx \Lambda_k |\tilde{\phi}_k|^2$;

$\Lambda_k \equiv 1 + \alpha_n k_\perp^2 + \alpha_{A_\parallel}$, $\alpha_n = k_y^2 / |\omega_k|^2$, $\alpha_{A_\parallel} \sim \left(\hat{\chi}_e k_\parallel / k_\perp^2\right)^2 \left(1 + k_y^2/|\omega_k|^2 - 2k_y\omega_{rk}/|\omega_k|^2\right)$

Here ω_{rk}^{lin} and γ_k^{lin} are given by Eq. (7) and the nonlinear modifications due to slow modes in frequency and growth are

$$\Delta\omega_q = \vec{k}_\perp \cdot \hat{z} \times \vec{\nabla}\tilde{\phi}_q - 0.5\beta[\vec{k}_\perp \cdot \hat{z} \times \vec{\nabla}\tilde{A}_{\parallel q}(\frac{\partial\omega_{1k}}{\partial k_\parallel})];\ \Delta\gamma_q = -0.5\beta[\vec{k}_\perp \cdot \hat{z} \times \vec{\nabla}\tilde{A}_{\parallel q}](\frac{\partial\gamma_{1k}}{\partial k_\parallel}) \quad (11)$$

$$\partial\omega_r / \partial k_\parallel = 0.5k_y[2 + \alpha_i(1-k_\perp^2)]\hat{\chi}_e k_\parallel / k_\perp^2 \gamma_0;\ \partial\gamma / \partial k_\parallel = -(1+k_\perp^2)\hat{\chi}_e k_\parallel / k_\perp^2 \quad (12)$$

Zonal flow and zonal fields instabilities: In the long wavelength limit i.e. $q_x^2 < 0.5\beta\hat{\chi}_e\gamma_q$ and wave number $q_x \neq 0$ and $q_y = q_z = 0$, the equations of ZFs and ZFLDs are

$$\left(\partial_t + \nu_i^{neo}\right)\tilde{\phi}_q = \left\langle \partial_x\tilde{\phi}_k \partial_y\tilde{\phi}_k \right\rangle - 0.5\beta\left\langle \partial_x\tilde{A}_{//k}\partial_y\tilde{A}_{//k} \right\rangle \approx \sum_k (1 - 0.5\beta\alpha_{A_\parallel})k_x k_y |\tilde{\phi}_k|^2 \approx \sum_k (1-0.5\beta\alpha_{A_\parallel})(k_x k_y |\omega_{rk}| / \Lambda_k)\delta N_q \quad (13)$$

$$\left(\partial_t + \hat{\eta}q_x^2\right)\tilde{A}_{//q} = -iq_x \sum_k \left(k_y\hat{\chi}_e k_\parallel / k_\perp^2\right)\left(\Lambda_k^* |\omega_{rk}| / \Lambda_k\right)\delta N_k \ , \quad (14)$$

Where $\Lambda_k^* \equiv \left(1 + k_y^2/|\hat{\omega}_k|^2 - 2k_y\hat{\omega}_{rk}/|\hat{\omega}_k|^2\right)$. After Fourier transform, the growth rates of ZF and ZFLD are

$$\gamma_q^\phi = q_x^2 \sum_k (1-0.5\beta\alpha_{A_\parallel})(k_x k_y^2 |\omega_{rk}| / \Lambda_k)(-\partial N_{0k} / \partial k_x) - \nu_i^{neo};$$
$$\gamma_q^{A_\parallel} = 0.5\beta q_x^2 \sum_k (k_y^2 \hat{\chi}_e k_\parallel / k_\perp^2)(\Lambda_k^* |\omega_{rk}| / \Lambda_k\gamma_k)(-\partial\gamma_k / \partial k_\parallel)N_{0k} - \hat{\eta}q_x^2. \quad (15)$$

The growth of zonal flows and zonal fields can be viewed as a cascade of energy from the small-scale to large-scale electrostatic potential (zonal flow) and magnetic field. Also note that the growth rate of zonal flow is derived from the modulation of the frequency of the background wave, whereas the zonal magnetic field results from the modulation of growth, and this can be viewed as a small-scale dynamo action.

In order to compare the growth rates of ZF and ZFLD in high–m DRBM regime, we use the following normalizations [1-2],

$$\hat{\omega}_k = \omega / \gamma_k^{ideal}, \quad \hat{m} = k_y L_0, \quad L_0 = \left(\rho_s^2 \gamma_k^{ideal} / \chi_{e\parallel} \langle k_\parallel \rangle^2\right)^{1/2}, \langle k_\parallel \rangle = 1/q_a R$$

$$\chi_{e\parallel} = c_e^2 / 0.5\nu_e, \quad q_a = rB_\varphi / RB_\theta, \quad \alpha_d = \left(\rho_s c_s \big/ L_o L_n \gamma_k^{ideal}\right), \quad \hat{\beta} = \beta q_a^2 R / L_n, \tag{16}$$

$$\gamma_k^{ideal} = c_s \left(2(1+\tau_i)/RL_n\right)^{1/2}, \quad \hat{\rho} = \rho_s / L_o$$

The frequency and growth of DRBM can be rewritten as

$$\hat{\omega}_{rk}\,(=\omega_{rk}/\gamma_{ideal}) \approx -0.5\alpha_i\alpha_d + \frac{\alpha_d\left(2+\alpha_i(1-\hat{m}^2\hat{\rho}^2)\right)}{4\hat{m}(1-\gamma_{flr})^{1/2}}; \quad \gamma_{flr} = \alpha_i^2 \hat{m}^2 \alpha_d{}^2 / 4$$

$$\hat{\gamma}_k (= \hat{\gamma}_k/\gamma_{ideal}) \approx \left((1-\gamma_{flr})^{1/2} - \gamma_{\chi-flr}\right); \quad \gamma_{\chi-flr} = (1+\hat{m}^2\hat{\rho}^2)/2\hat{m}^2; \quad |\hat{\omega}_k| = \hat{\omega}_{rk}{}^2 + \hat{\gamma}_k{}^2$$

The zonal flow and zonal field growth rates are

$$\hat{\gamma}_q^\phi \approx \left(q_x/k_y\right)^2 \left(\alpha_d \hat{m}^2\right)^2 (1-\gamma_{flr})^{-1/2}\left(1-\hat{\beta}\hat{m}^{-4}(1+\tau_i)F(\alpha_d,\hat{\omega}_k)\right) \times |e\delta\phi_k L_n / L_0 T_e|^2 - \hat{\nu}_i^{neo}$$

$$\hat{\gamma}_q^{A_\parallel} \approx \hat{\beta}(1+\tau_i)\left(\alpha_d q_x L_n / \hat{m}\right)^2 (1+\hat{m}^2\rho^2) F(\alpha_d,\hat{\omega}_k) \big/ (1-\gamma_{flr})^{1/2} \times |e\delta\phi_k / T_e|^2 - \hat{\eta}_0$$
$$\approx \left(q_x/k_y\right)^2 \hat{\beta}\alpha_d^2 (1+\tau_i) F(\alpha_d,\hat{\omega}_k)(1-\gamma_{flr})^{-1/2} \times |e\delta\phi_k L_n / L_0 T_e|^2 - \hat{\eta}_0$$

Where $F(\alpha_d,\hat{\omega}_k) = 1 + \alpha_d^2 \hat{m}^2 / |\hat{\omega}_k|^2 - 2\alpha_d \hat{m}\hat{\omega}_{rk} / |\hat{\omega}_k|^2$, $\hat{\gamma}_q^{A_\parallel} = \gamma_q^{A_\parallel} / \gamma_{ideal}$ and $\hat{\gamma}_q^\phi = \gamma_q^\phi / \gamma_{ideal}$, $\hat{\nu}_i^{neo} = \nu_i^{neo} / \gamma_{ideal}$ is the neoclassical ion collision rate. Note that for $\hat{m} > 1$ and $\hat{\beta} < 1$, the high-m DRBM is the leading unstable mode in a collisional edge [2], and the growth rate of zonal flow is larger than the zonal fields growth, whereas as in the opposite limit, zonal fields dominate over zonal flow and in this case the growth of high-m DRBM is also weak. Thus the dynamics of ZFs in the saturation of edge fluctuations is important. Also note that the ZFs are decoupled from the ZFLDs.

Transport estimates: We now wish to make an estimate of the transport due to DRBM in the edge of a tokamak. These fluctuations typically have large poloidal mode number $\hat{m} > 1$ and therefore grow to large amplitudes saturating only due to zonal flow instabilities. From a predator-prey model, the saturation amplitude of DRBM and diffusion coefficient can be written as

$$\frac{e\delta\phi_k}{T_e} \approx \frac{(1+\tau_i)q_a^2 R}{L_n} \frac{\nu_e (\nu_i)^{1/2}}{(\gamma_{ideal})^{1/2}\Omega_e}; \qquad D \approx \frac{1}{(\hat{\omega}_{rk}^2 + \hat{\gamma}_k^2)^2}(1+\tau_i)\nu_e \nu_i^{neo} \rho_e^2 \frac{q_a^2 R}{\gamma_{ideal} L_n} \tag{17}$$

Tertiary instability - the Kelvin-Helmholtz instability:

Here we examine the conditions under which the strong velocity shear and magnetic shear associated with ZFs and ZFLDs may lead to their break-up due to a tertiary instability (e.g. K-H instability). In such conditions the dominant saturation mechanism for primary turbulence by the ZFs will become ineffective. For simplicity we consider a 1-D zonal flow and zonal field, which are independent of y coordinate: $\phi_q(x) = \phi_0 \cos qx$; We use the Floquet technique[9-10] and take the K-H instability perturbation to be $\delta\phi = \sum \phi_n \sin\left((K_x + nq)x + K_y y\right) e^{\gamma_{KH} t}$.

Considering zonal waves as the equilibrium background for K-H wave, the simple form of linearized K-H wave equations can be obtained from Eq. (17),

$$\gamma_{KH} \nabla_\perp{}^2 \tilde{\phi}_{KH} - (2\beta^{-1}/\gamma_{KH}) \nabla_\parallel^2 \nabla_\perp{}^2 \tilde{\phi}_{KH} = -[\tilde{\phi}_\pm, \nabla_\perp{}^2 \tilde{\phi}_q] - [\tilde{\phi}_q, \nabla_\perp{}^2 \tilde{\phi}_\pm] \tag{18}$$

For small ZF amplitudes, we truncate K-H wave equation by keeping the mode coupling among three adjacent modes (i.e. $n = 0, \pm 1$), and then the growth rate of K-H instability can be written as

$$\gamma_{KH}^2 = 0.25 |\phi_{0q}|^2 q^2 K_y^2 \frac{q^2 - K_\perp^2}{K_\perp^2} \left(\frac{K_\perp^2 + 2qK_x}{K_\perp^2 + q^2 + 2qK_x} + \frac{K_\perp^2 - 2qK_x}{K_\perp^2 + q^2 - 2qK_x} \right) - 0.25\beta |A_{0q}|^2 q^2 K_y^2 - 2K_\parallel^2 / \beta \qquad (19)$$

Note that the K-H instability is restricted to scales $k_\perp^2 > q^2 > K_\perp^2$. To estimate the threshold of K-H instability, we simplify by maximizing the driving term, which requires $K_x \to 0$, $q^2 > K_y^2$ and $K_\parallel \sim \hat{s} / qR$, where $\hat{s}$ is the magnetic shear. The growth rate of K-H mode as a function of α_d and $\hat{\beta}$ parameters is then

$$\hat{\gamma}_{KH}^2 = 0.5 q^2 K_y^2 k_y^{-4} \alpha_d^2 \hat{m}^4 \left(|e\phi_{0q} L_n / T_e L_0|^2 - (\hat{\beta}\varepsilon_n / 4q_a^2) |eA_{0q} L_n / T_e L_0|^2 \right) - \hat{s}^2 / \hat{\beta}(1+\tau_i)$$

Note that the ZFLDs provide an additional effective magnetic shear to stabilize the tertiary instability. For simplicity we neglect the ZFLDs effects on tertiary modes, and a predator-prey model yields $\phi_{0q} \approx \gamma_k / \alpha_0$, where $\alpha_0 \sim q^{-2} K_y^2 \alpha_d^2 \hat{m}^4$.

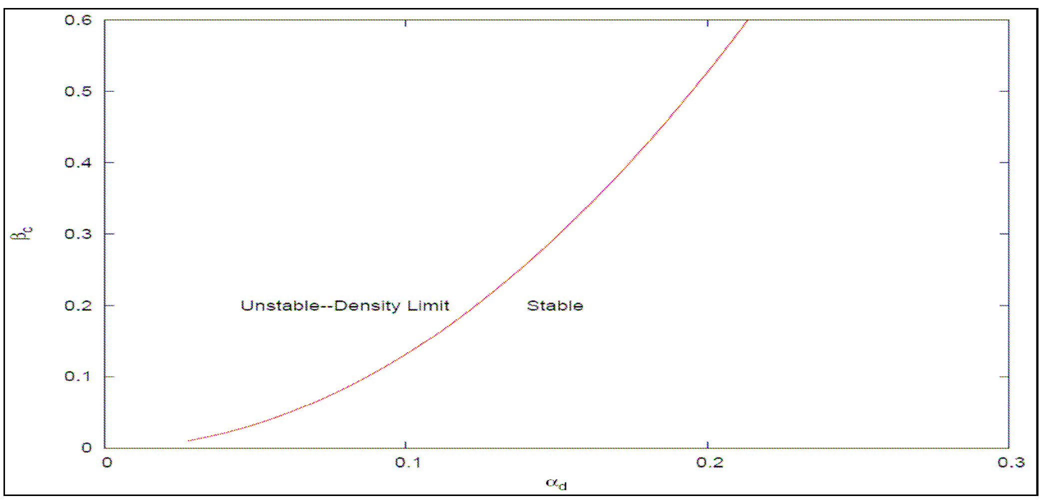

Figure 1

The threshold of tertiary mode (K-H instability) i.e., $\gamma_{K-H} \approx 0$ could determine the density limit. Making use of this condition $\gamma_{K-H} \approx 0$, we get the dependence of critical density n_{cr} on the basic plasma parameters as:

$$n_{cr} \sim I^{0.93} R^{0.58} A_i^{0.12} \hat{s}^{0.47} a^{-1.86} B^{-0.19}.$$

Here we have used the following conditions[11]: (1) the convective thermal flux entering into edge region is dominant and acts like a source for the edge plasma. The convective thermal flux from core is defined as $nTD^{GB} / L_n \approx Const$. (2) density scale length $L_n \sim 1/n_e$. Figure 1 shows the density limit boundary in the $\hat{\beta} - \alpha_d$ plane.

Geodesic acoustic modes: GAMs are the $m = n = 0, \quad k_r \neq 0$ perturbation in potential fields and $n = 0, \quad m = 1, \quad k_r \neq 0$ perturbations in density, parallel flows, and pressure fields. The dispersion relation of electrostatic GAM is

$$\Omega_G{}^2 - k_\parallel^2(1+\tau_i) - (1+\tau_i)\varepsilon_n^2 < \sin^2\theta > \approx -q_x^2 k_y^2 [\Omega^2 - k_\parallel^2(1+\tau)/\Omega_G \bar{\Omega}] |\tilde{\phi}_k|^2 \quad (20)$$

For $\Omega_G - q_x V_{gx} < \gamma_k$, and taking the nonlinear term perturbatively, the frequency and growth rate of GAM are

$$\hat{\Omega}_{rG} \approx (2q_a^2+1)^{1/2}(\sqrt{\varepsilon_n}/2q_a); \quad \hat{\gamma}_G \approx [q_a \alpha_d \hat{m} q_x L_n/(1+2q_a)]^2 |e\delta\phi_k / T_e|^2 \quad (21)$$

The ratio of the growth rates of GAMs and zonal flow gives an enhancement factor:

$$\hat{\gamma}_G / \hat{\gamma}_q^\phi \approx \left[\left(q_a / 1+2q_a\right)^2 \Big/ \left(1 - \hat{\beta}\hat{m}^{-4}(1+\tau_i)F(\alpha_d, \hat{\omega}_k)\right) \right] \quad (22)$$

Note that when the factor $\left(1 - \hat{\beta}\hat{m}^{-4}(1+\tau_i)F(\alpha_d, \hat{\omega}_k)\right)$ begins to approach unity, concurrently the growth of zonal flow diminishes and the growth rate of GAM instability enhances.

Coherent nonlinear structures of ITG wave: For $C > 1$, we study the trapping of ITG mode turbulence in secondary ZFs, which produce nonlinear coherent structures with saturated ITG mode turbulence and the ZFs nonlinearly sustaining each other with trapped and untrapped distributions of primary turbulence. The basic requirements for trapping are: (1) the short-scale mode is dispersive in nature; (2) the bounce frequency of trapped ITG quasi-particle in ZF potential is larger than the growth rate of ZFs i.e., $\nu_i^{neo} < \gamma_{ZF} < \omega_B$, ω_B is the bounce frequency of trapped particle. We look at the stationary solutions of the coupled ITG-ZF system described by the WKE and vorticity equation

$$\partial N_k / \partial t + V_{gx} \cdot (\partial N_k / \partial x) - (\partial \omega_E / \partial x) \cdot (\partial N_k / \partial k_x) \quad (23)$$

$$\left(\partial_t + \nu_i^{neo}\right) \nabla_x^2 \tilde{\phi}_q = \nabla_x^2 \int \partial^3 k \, k_x k_y (1+\tau_i+\Lambda_0) N_k / \Delta_* \quad (24)$$

Eq. (23) can be readily integrated to give the constant of motion: $W = K_x^2 + f(x)$. Here $K_x = (k_x + U/4bk_y); \quad f(x) = v + \tau_i v'' - U^2/16b^2k_y^2, \quad v = \partial_x \phi_q, \quad X = x - Ut$. The bounce frequency of ITG mode near the minima of ZFs shear layer can be obtained from the characteristic ray equations for dispersive ITG mode,

$$\partial x / \partial t = V_{gx} \approx -2k_x k_y b; \quad \partial k_y / \partial t = 0; \quad \partial k_x / \partial t = -\partial \omega_k / \partial x \approx -k_y \partial^2 \phi_q / \partial x^2. \quad (25)$$

Here $\hat{\omega}_{rk} \approx a - bk_\perp^2$, $\gamma_k \approx k_y \sqrt{\tau_i \varepsilon_n (\eta_i - \eta_{th})}$, $b = 0.5[1+\tau_i(1+\eta_i) - \varepsilon_n(1+5\tau_i\varepsilon_n/3)]$.

Note that ϕ_q is oscillating in nature and for choice of ϕ_q, we can solve the above characteristic ray equations. Let $\phi_q = \phi_0 \sin x \sim \phi_0(x - x^3/6)$ and with boundary condition $k_x \sim \partial k_x / \partial t = 0$ at $t = 0$. The typical bounce frequency in dimensional form

$$\omega_b = \omega_* \sqrt{1+\tau_i(1+\eta_i) - \varepsilon_n(1+5\tau_i/3) \left| e\phi_{0q} L_n / \rho_s \right|}; \; \omega_* = k_y \rho_s c_s / L_n. \quad (26)$$

The solution of stationary Eq. (23-24) can be obtained from nonlinear self-consistency condition:

$$(\mu \nabla_x^2 - \nu_i^{neo} + U \nabla_x) v = -0.5 \bar{a} \nabla_x \left[\int_f^{f_m} dW J(W) N_T(W) + \int_{f_m}^{\infty} dW J(W) N_U(W) \right] \quad (27)$$

We choose trapped $N_T(W)$, and untrapped distribution of quasi-ITG particle as

$$N_U = N_0[1+(W-f_m)^2/\Delta^2]^{-1}; \quad W > f_m$$
$$= N_0[1+\varepsilon(f_m - W)^{1/2}/\Delta^{1/2}]; \quad f < W < f_m \tag{28}$$

Here ε is the fraction of trapped particles. For dissipation, $(\nu, \mu) = 0$ and by considering nonlinear term up to $v^{3/2}$, Eq. (27) yields

$$\sigma_0 V'' - (U-\sigma_0)V + \sigma_1 V^{3/2} = 0; \quad f_m - f = V \tag{29}$$

Here $\sigma_0 = 0.5\bar{a}\Delta(1-\bar{b}k_y^2)$, $\sigma_1 = \Delta\varepsilon a(1-bk_y^2)/3$, $\bar{a} = W_{*T}ak_y/1+W_{*T}$, $\bar{b} = (b/a)+1/(1+W_{*T})$, $\sigma_1 = \Delta\varepsilon a(1-bk_y^2)/3$. An exact solution of Eq. (28) gives the radially propagating solitary pulse,

$$V = [5(U-\sigma_0)/4\sigma_1]^2 S\,\mathrm{ec}\,h\,[(U-\sigma_0)(x-Ut)/\sigma_0]. \tag{30}$$

The modulational instability of ZFs, ZFLDs, and GAMs in the background of DRBM driven turbulence has been studied. It is shown that the short-scale and long-scale ZFs, ZFLDs, and tertiary modes can co-exist (predator-prey). A quasi-steady state of such a system could be determined by the comparison of their relative growth rates to the neoclassical collisional damping rate. An L-mode could be thought of as a state where the growth of ZFs is less or comparable to ion neoclassical collisional rate. (2) H-mode can be attained when the ZFs growth rate is larger than neoclassical collisional damping, as well as the ZF is stable to the tertiary wave - KH instability. (3) Greenwald density limit could be hit if the ZF is unstable to the tertiary mode, if $\gamma_{K-H} \cong 0$. From the "Predator-Prey" model for the self-interaction of ZFs and short-scale background DRBM, the transport in a collisional edge is estimated. From the threshold condition for the tertiary modes, a dependence of critical density on other basic plasma parameters has been derived.

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