Diamagnetic GAM Drive Mechanism

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Abstract. GAMs oscillate between states of strong rotation and up-down asymmetric plasma compression. Hence, at first glance the natural turbulent drive (or damping) mechanisms for them is either a direct boost of the rotation – via Reynolds stress – or the creation of asymmetric pressure distributions – via transport. However, an up-down asymmetric pressure can also be created by the divergence of the diamagnetic drifts, if there is a perturbation in the diamagnetic drift velocity, i.e., of the overall pressure gradient. That in turn may fluctuate due to any modulation of the flux surface averaged turbulent transport. On the other hand, a modulation of the turbulent transport due to the GAMs themselves is expected to happen in the tokamak edge, and has been observed early on in simulations and recently in many experiments. In turbulence simulations for edge parameters, the described effect tendencially is a strong driver of the GAMs of equal importance to the other two. As a striking consequence, the coupling of diamagnetic velocity and GAM can produce propagating fronts of high flow velocity and transport, which closely resemble avalanches – without necessity of a critical gradient. The diamagnetic flow drive is strong enough to advance the flow and transport layer in radial direction – although the linear dispersion relation would just result in a localized oscillation! An interesting consequence of the diamagnetic drive mechanism is that it offers the possibility of direct excitation of GAMs by resonantly modulated external heating (replacing turbulent transport with heating power). If the GAMs are detected by Doppler reflectometry, the achievable efficiency is certainly enough for diagnostic purposes such as to *actively* probe the GAM frequencies or to measure the turbulence response to the GAMs. Particularly exciting however is the prospect of a way to *artificially* set up a GAM pattern to control the transport.

1. Introduction

Geodesic Acoustic Modes (GAM), poloidal flows oscillating at the characteristic acoustic frequency of a tokamak or stellarator, are an ubiquitous edge plasma phenomenon in magnetic fusion devices [1, 2]. In recent years they have dramatically gained experimental interest and are candidates for applications ranging from plasma diagnostics [3] to transport control [2].

GAMs and the somewhat better known stationary Zonal Flows arise as the two linear eigenstates from the coupling of perpendicular plasma rotation and parallel sound waves by magnetic inhomogeneities such as due to toroidal curvature. Both have virtual no radial velocity component whence they are in practice completely stable against any radial pressure gradients. Although turblence driven stationary Zonal Flows have been theoretically predicted somewhat earlier than the GAMs, the latter were detected first in experiments, due to their clear signature of a rather well defined frequency.

GAMs oscillate between states of poloidally homogeneous rotation and (for a tokamak) up-down antisymmetric plasma compression. Hence, at first glance the natural turbulent drive (or damping) mechanisms for them seem to be either a direct boost of the rotation – via Reynolds stress – or the creation of antisymmetric pressure distributions – by vertical oscillations of the turbulent heat transport – which may both be synchronised with the flow oscillation by its shearing action.

2. Mechanism

However, an up-down antisymmetric pressure can also be created more indirectly by the turbulence if it perturbs the ion diamagnetic drift velocity or more accurately the diamagnetic heat flux, e.g., by local flattening of the overall radial pressure gradient due to turbulent transport. Due to the inhomogeneous magnetic field, the perturbed ion diamagnetic flow and heat flow will exhibit an up-down antisymmetric divergence which creates a pressure perturbation exciting the GAM. (While the diamagnetic flows of the electrons have exactly the opposite divergence, their asymmetries are instantaneously erased due to the much greater electron mobility along the magnetic field lines.)

This requires a diamagnetic velocity modulation in resonance with the GAMs, and thus a modulation of the flux surface averaged turbulent transport in the proper phase relation with the oscillating flow. On the other hand, a modulation of the turbulent transport due to the GAMs themselves is expected to happen in the tokamak edge, and has been observed early on in simulations [4] and recently in many experiments [5].

In the simulations [4] it was found that the turbulent transport is modulated by the shear flow such that it is essentially proportional to the local flow velocity in electron diamagnetic direction. This may be described roughly by the empirical relation

$$\delta Q = \gamma\partial_r^2(Q - \alpha v_\theta + \beta v_{di}). \tag{1}$$

with appropriate empirical constants α, β, γ. The form of this relation is severely restricted by Gallilean invariance in θ direction of the fundamental equations, mirror symmetry with respect to the minor radius r and the fact that the poloidal $\boldsymbol{E}\times\boldsymbol{B}$ velocity v_θ is only important through its effect on the phase velocity. For example, the relation cannot directly involve v_θ but only its derivatives, v_θ. The contribution from the poloidal ion diamagnetic velocity v_{di} occurs, since the phase velocity is the difference of v_θ and a mode specific constant times v_{di}. γ is typically rather large, so that essentially $\delta Q \approx \alpha v_\theta - \beta v_{di}$ as for the relevant radial wavenumbers of GAMs $k_r^2\gamma \lesssim 1$. For simplicity the contribution from the diamagnetic velocity outside the radial derivative operator has been neglected even though in principle the transport for very low wavenumbers is reduced if the ion diamagnetic velocity goes up, i.e., if the gradient decreases.

Neglecting neoclassical transport one obtains from the radial heat transport balance a modulation of the local ion diamagnetic velocity equal to

$$-i\omega\delta v_{di} = -i\omega\partial_r\delta p_i = -\frac{2}{3}\partial_r^2\delta Q = -\frac{2\alpha}{3}\partial_r^2 v_\theta + \frac{2\beta}{3}\partial_r^2 v_{di}, \tag{2}$$

$$\delta v_{di} = \frac{2i\alpha k_r^2 v_\theta}{3\omega + 2i\beta k_r^2} \tag{3}$$

with the dimensionless units $L_\perp = \rho_i, t_0 = R/(2\sqrt{T_0/m_i})$ and the fluctuation variables

$$n = \frac{\delta n\rho^*}{n_0}, \qquad T_i = \frac{\delta T_i\rho^*}{T_0}, \qquad \phi = \frac{e\delta\phi\rho^*}{T_0} \tag{4}$$

The fluid ion density and temperature equation equations for the GAMs may be written as

$$\dot{n} - \Delta\left(\dot{\phi} + \dot{n} + \dot{T}_i\right) - C(\phi + n + T_i) = 0 \tag{5}$$

$$\dot{T}_i - \frac{2}{3}\Delta\left(\dot{\phi} + \dot{n} + \frac{7}{2}\dot{T}_i\right) - \frac{2}{3}C\left(\phi + n + \frac{7}{2}T_i\right) + \frac{2}{3}\partial_r Q = 0, \tag{6}$$

where for simplicity of the following argument, in (6) parallel sound waves and heat conduction have been omitted. The geodesic curvature operator

$$C \equiv \left(\nabla \times \frac{\boldsymbol{b}}{B}\right) \cdot \nabla \approx 2\left(\frac{\boldsymbol{b}}{B} \times \boldsymbol{\kappa}\right) \cdot \nabla, \qquad \boldsymbol{\kappa} = \partial_{\parallel} \boldsymbol{b}, \qquad \boldsymbol{b} = \frac{\boldsymbol{B}}{B} \tag{7}$$

computes the divergence of the $\boldsymbol{E} \times \boldsymbol{B}$ and diamagnetic flows from ϕ and n, T_i, respectively. In circular geometry the geodesic curvature terms are simply $C = \sin\theta\partial_r$. The finite larmor radius (FLR) (or diamagnetic) heat flux requires the factor $7/2$ instead of 1 in front of the second and third occurance of T_i in (6), which is shown below to be essential for the diamagnetic drive mechanism. The electrons may simply be taken to be adiabatic,

$$n = \phi - \phi_0 \tag{8}$$

where $\phi_0 \equiv \langle\phi\rangle$ is the flux surface average of the electric potential and ion and electron density are equal due to quasineutrality. Assuming $k^2\rho_i^2 \ll 1$ (as is corroborated by turbulence simulations), the Laplace operators in (5,6) can be neglected, except when taking the flux surface average of (5), since then $n_0 \equiv \langle n\rangle = \langle\phi\rangle - \langle\phi\rangle = 0$ due to equation (8), and the Laplacian contains the *only* time derivatives in the equation. The flux surface average of (5) reads

$$-\Delta\left(\dot{\phi}_0 + \dot{T}_{i,0}\right) - \langle C(\phi - \phi_0 + n + T_i - T_{i,0})\rangle = 0 \tag{9}$$

$$\Leftrightarrow -\Delta\left(\dot{\phi}_0 + \dot{T}_{i,0}\right) - \langle C(2n + T_i - T_{i,0})\rangle = 0. \tag{10}$$

Integrating this equation over r, noting that $\Delta \equiv \partial_r^2$ and $v_\theta = \partial_r\phi_0$, $v_{di} = \partial_r(n_0 + T_{i,0}) = \partial_r T_{i,0}$ yields

$$\dot{v}_\theta + \dot{v}_{di} = -\langle(\sin\theta)(2n + T_i - T_{i,0})\rangle. \tag{11}$$

The θ-dependence of the fluctuations can be obtained by combining (5) and (6), neglecting now the Laplacian and taking into account that Q is independent of θ,

$$2\dot{n} + \dot{T}_i - \dot{T_{i,0}} - \frac{1}{3}C(8\phi + 8n + 13T_i) = 0. \tag{12}$$

$$\Leftrightarrow 2\dot{n} + \dot{T}_i - \dot{T_{i,0}} - \frac{1}{3}C(8\phi_0 + 16n + 13T_i) = 0. \tag{13}$$

Using the additional assumption, that $\omega/k_r \gg 1$, i.e., that the phase velocity is much larger than the curvature drift velocity, one can neglect the curvature term acting on n and $T_i - T_{i,0}$ in (13) and write

$$2\dot{n} + \dot{T}_i - \dot{T_{i,0}} = \frac{1}{3}C(8\phi_0 + 13T_{i,0}) = \frac{1}{3}\sin\theta(8v_\theta + 13v_{di}). \tag{14}$$

Inserting (14) into the time derivative of eq. (11) and carrying out the flux surface averages results in

$$\ddot{v}_\theta + \ddot{v}_{di} = -\frac{1}{6}(8v_\theta + 13v_{di}) \tag{15}$$

With $\omega_{GAM} = 2/\sqrt{3}$ in this simple model, the GAM velocity obeys thus the equation

$$\omega^2(v_\theta + \delta v_{di}) = \omega_{GAM}^2\left(v_\theta + \frac{13}{8}\delta v_{di}\right), \tag{16}$$

resulting in the dispersion relation

$$\omega^2\left(1 + \frac{2i\alpha k_r^2}{3\omega + 2i\beta k_r^2}\right) = \omega_{GAM}^2\left(1 + \frac{13}{8}\frac{2i\alpha k_r^2}{3\omega + 2i\beta k_r^2}\right). \tag{17}$$

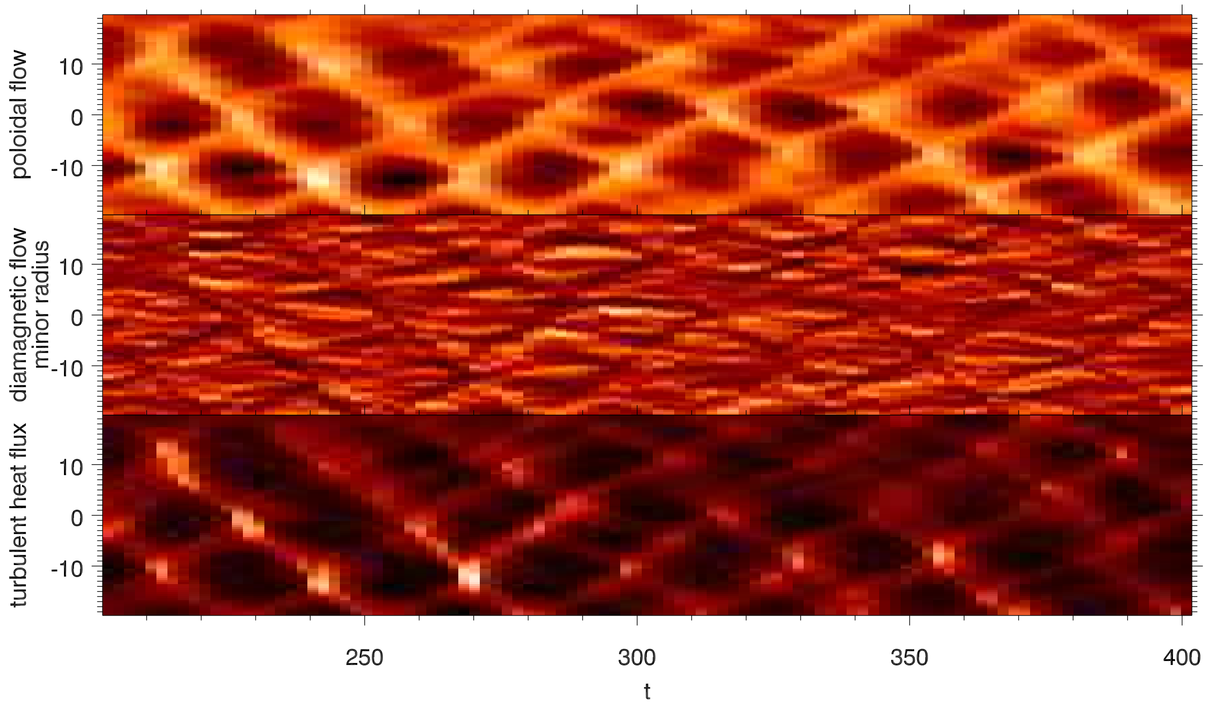

Figure 1: Color coded plots of GAM poloidal flow velocity (top), ion diamagnetic velocity (middle), turbulent ion heat flux (bottom) versus time and minor radius from a fluid turbulence simulation for parameters from the transitional regime [4]. $k_{r,max} = 0.4$ taking $\beta \sim 3\chi$, where χ is the heat diffusivity, which corresponds to a preferred GAM-Wavelength of about 20 in the units of this figure.

Note that without the FLR heat flux in eq. (6) the factor $13/8$ in this equation were in fact be one and the diamagnetic terms would cancel exactly. The described GAM drive mechanism indispensibly requires the FLR heat flux.

To lowest order in α the dispersion relation yields

$$\omega \approx \omega_{GAM} + \frac{5}{8}\left(i\frac{3\alpha k_r^2}{9\omega_{GAM}^2 + 4\beta^2 k_r^4} + \frac{2\alpha\beta k_r^4}{\omega_{GAM}(9\omega_{GAM}^2 + 4\beta^2 k_r^4)} \right). \quad (18)$$

The imaginary component indicates growth of the GAMs provided that α is positive, i.e., that local poloidal flows in electron (ion) diamagnetic direction are accompanied by maxima (minima) of turbulent transport (as was observed computationally [4] and experimentally [6]).

The growth rate in Eq. 18 exhibits a maximum at a wavenumber $k_{r,max} = \sqrt{3\omega/(2\beta)}$ depending on the sensitivity β of the turbulence to the gradients (essentially the differential diffusivity, about 2-3 times the turbulent diffusivity). Nonlinear effects beyond this toy model, the other two turbulent drive/damping terms mentioned above, and dissipation likely will reduce the growth rates overall while still basically conserving the wavelength scaling.

Fig. 1 shows a flux surface averaged flow velocity, ion diamagnetic velocity, ion heat flux versus time and minor radius from a fluid turbulence simulation for parameters from the transitional regime [4]. Note the characteristic diamagnetic-velocity double-layers caused by the ion heat flux modulation in phase with the GAMs. About 30% of the GAM-drive in this case stem from the diamagnetic velocity modulation.

Whether the above mechanism is an effective driver of the GAMs depends on the strength of

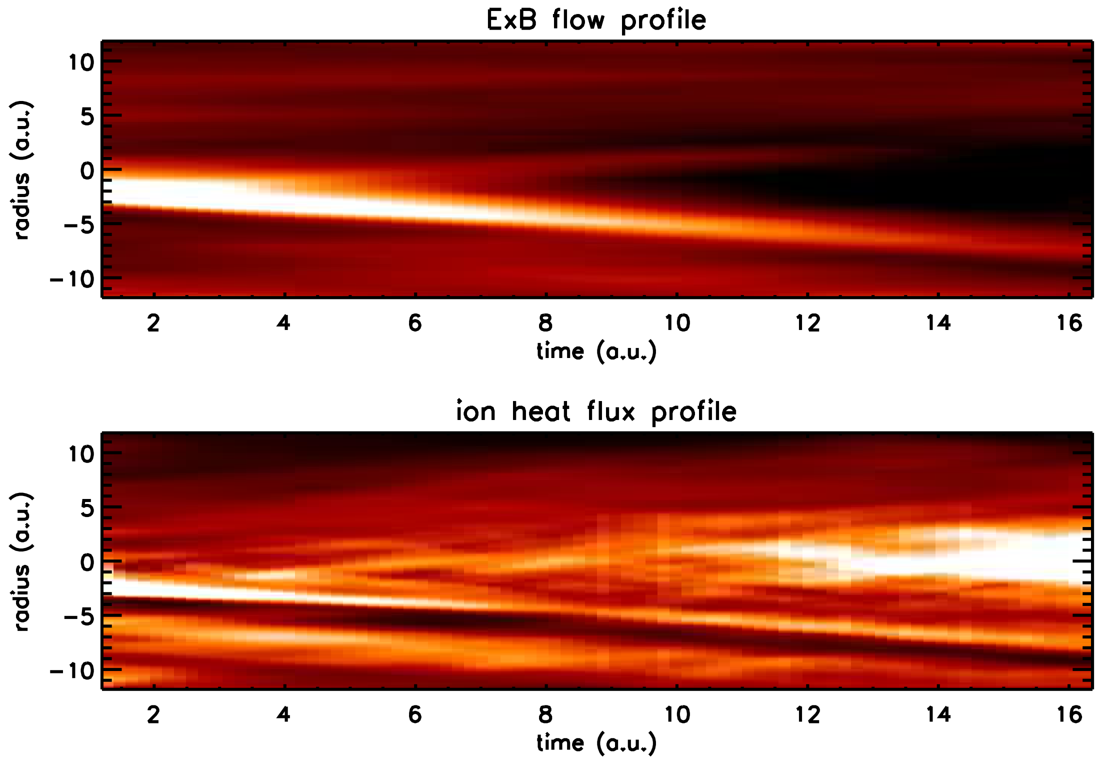

Figure 2: plots of flow (top) and turbulent heat flux (bottom) for the time evolution of an initially isolated single GAM peak

the transport modulation, the geometry controlling the importance of the diamagnetic drifts, and the ratio of turbulence time scales and GAM frequency. It is however persuasive that the estimate of the GAM wavelength derived from the above argument agrees with the observed one. In turbulence simulations for edge parameters, the described effect tendencially is a strong driver of the GAMs of equal importance to the other two.

3. Consequences

As a striking consequence, the coupling of diamagnetic velocity and GAM can produce propagating fronts of high flow velocity and transport, which closely resemble avalanches – without necessity of a critical gradient: Fig. 2 shows the time evolution of an initially isolated single GAM peak (with turbulence) for identical background parameters as Fig. 1. The diamagnetic flow drive is strong enough to advance the flow and transport layer in radial direction (the preferred wave-number derived above fixes the phase velocity considering that the GAM frequency is mostly determined by linear physics) – although the linear dispersion relation would just result in a localised oscillation!

An interesting feature of the diamagnetic drive mechanism is that it offers the possibility of direct excitation of GAMs by resonantly modulated external heating (replacing turbulent transport in (1) with heating power). If the GAMs are detected by Doppler reflectometry, the achievable efficiency is certainly enough for diagnostic purposes such as to *actively* probe the GAM frequencies or to measure the turbulence response to the GAMs. Particularly exciting however is the prospect of a way to *artificially* set up a GAM pattern to control the transport.

References

[1] P. H. Diamond, S.-I. Itoh, K. Itoh, T.S. Hahm, Plasma Phys. Control. Fusion **47**, R35 (2005)

[2] K. Hallatschek, Plasma Phys. Control. Fusion **49**, B137 (2007)

[3] S.-I.Itoh, et al., Plasma Phys. Control. Fusion **49**, L7 (2007)

[4] K. Hallatschek, D. Biskamp, Phys. Rev. Lett. **86**, 1223 (2001)

[5] G. D. Conway, et al., Plasma Phys. Control. Fusion **50**, 055009 (2008)

[6] A. Fujisawa, et al., Nucl. Fusion **47**, S718 (2007)