Shear Alfvén Wave Continuous Spectrum in the Presence of a Magnetic Island*

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Abstract. The continuous spectrum of shear Alfvén waves is calculated for a finite-beta tokamak equilibrium in the presence of a finite-size magnetic island. The beta-induced Alfvén Eigenmodes continuum accumulation point (BAE-CAP) is found to be positioned at the separatrix flux surface, while the frequency remains the same. The most remarkable novel feature is the presence of new CAPs at the island O-point, which give rise to gaps in the continuous spectrum. This could make the existence of new magnetic-island induced Alfvén Eigenmodes (MiAE) possible, excited via wave-particle resonances if the island is sufficiently wide with respect to the mode radial localization. Experimental data of modes observed in Frascati Tokamak Upgrade (FTU) are presented, which are consistent with theoretical scalings of the MiAE-CAPs. Due to the frequency dependence on the magnetic island size, the feasibility of utilizing this scaling as a novel magnetic island diagnostic is also discussed.

1. Introduction Shear Alfvén waves (SAW) are electromagnetic plasma waves propagating with the characteristic Alfvén velocity $v_A = B/\sqrt{4\pi\varrho}$ (B is the magnetic field and ϱ the mass density of the plasma). In fusion plasmas, fast ions in the MeV energy range have velocities comparable with the typical Alfvén speed and can therefore resonantly interact with SAW. In addition, SAW group velocity is directed along the magnetic field line and, therefore, fast ions can stay in resonance and effectively exchange energy with the wave [1, 2]. SAW in a nonuniform equilibrium experience energy absorption (*continuum damping* [3, 4]), due to singular structures that are formed at the resonant surfaces of the SAW continuum. Due to nonuniformities along the field lines in toroidal geometry, gaps appear in the SAW continuous spectrum [5]. The mechanism is similar to that which creates forbidden energy bands for an electron traveling in a periodic lattice. Two types of collective shear Alfvén instabilities exist in tokamak plasmas: Energetic Particle continuum Modes (EPM) [6], with frequency determined by fast particle characteristic motions, and discrete Alfvén Eigenmodes (AE), with frequency inside SAW continuum gaps [7]. The former can become unstable provided the drive exceeds a threshold determined by the continuum damping absorption; the latter, on the other hand, has a generally lower instability threshold, being practically unaffected by continuum damping [1, 2]. For this reason, the importance of understanding the continuous spectrum structure is clear if one faces the stability problem of a tokamak and its potential impact on reaching the ignition condition.

The SAW nonlinear dynamics is a topic still rich of open issues. In the special case of uniform plasmas a peculiar state exists, called the *Alfvénic state*, which arises whenever we can assume ideal MHD, plasma incompressibility and the validity of the Walén relation ($\delta v/v_A = \pm\delta B/B$, where δv and δB are the perturbed velocity and field) [8]. In such a case, the nonlinear effects due to Maxwell and Reynolds stresses cancel and give a self-consistent nonlinear state. On the other hand, in nonuniform tokamak plasmas, with non-ideal effects such as resistivity or finite plasma compressibility, the Alfvénic state conditions are broken and SAW are characterized by an effective nonlinear dynamics. As a consequence, the SAW continuous spectrum can be modified by the interaction with low-frequency MHD fluctuations, such as magnetic islands, which are formed when the original sheared equilibrium magnetic field lines break due to non ideal effects (in particular finite resistivity) and reconnect with different magnetic topology [9]. Since the typical island frequency and growth rate are much lower than the SAW oscillation frequency, we can assume that the equilibrium magnetic field is given by the usual tokamak axisymmetric field plus a quasi-static helical distortion due to the magnetic island. We derive our fluid theoretical description of the SAW continuum structure in the presence of a finite size magnetic island [10] in finite-β plasmas, keeping into account only toroidicity effects due to the geodesic curvature, which are responsible for the beta-induced Alfvén Eigenmodes (BAE) gap in the low frequency SAW continuous spectrum [11, 12, 13]. The local differential equation yielding the nonlinear SAW continuum structure is solved numerically, with a shooting method code, in the whole spatial range of interest (both outside and inside the island), and analytically, far from the island separatrix, where the equilibrium quantities can be approximated in simple form.

As suggested by the magnetic field line helicity behavior [14, 15, 16], in an equilibrium with a magnetic island the separatrix flux surface plays an important role, hosting the BAE-CAP, that - without island - was positioned at the rational surface. Several branches of nonlinear SAW continuous spectrum stem from the BAE-CAP, parameterized by the SAW toroidal mode number n. Moreover, we find that the degeneracy of even and odd modes is removed by inhomogeneities along the field lines, causing a splitting between the continuous spectrum branches of even and odd modes. Outside the island, the branches of the nonlinear SAW continuum asymptotically approach the linear space dependence, typical of the SAW continuum in a sheared magnetic field and in the absence of the island; meanwhile, inside the island, they reach a finite value at the $O-$point, depending on n. These several CAPs at the $O-$point are peculiar to the presence of the island and have similar radial structures of those generated by reversed magnetic shear [17, 18]. This result has potential implications in explaining stability properties of tokamak plasmas in presence of magnetic islands. In fact, new magnetic-island induced Alfvén Eigenmodes (MiAE) could be excited inside a magnetic island if the thermal or energetic components of the plasma provide sufficient free energy for driving the mode.

Modes in the BAE frequency range have been observed in the Frascati Tokamak Upgrade (FTU) [15, 16, 19], in the presence of an (m, n) = (-2,-1) magnetic island, m indicating the poloidal mode number. A theoretical analysis has showed that these modes can be interpreted as BAE modes, when thermal ion transit resonances and finite ion Larmor radius effects are accounted for, with good agreement of measured and calculated frequencies in the small magnetic island amplitude limit [20]. In fact, their measured frequencies were found to depend on the magnetic island amplitude as well, consistently with the dependence of the MiAE-CAP on the magnetic island field strength. The modes were observed only when the magnetic island size was over a certain critical threshold. Similar observations have been reported by TEXTOR [21, 22]. Due to the dependence of the MiAE-CAP frequency on mode numbers and the magnetic island size, the possibility of using this scaling as novel magnetic island diagnostic is an attractive option. Analysis of Mirnov coil and soft X-Ray data are presented along with a discussion of the possibility of inferring from them the mode frequency, radial structure and localization on the basis of our present theoretical analysis.

2. Theoretical model

The fluid equation for SAW in a tokamak with quasi-static magnetic islands ($\omega_{isl}/\omega \ll 1$) and finite-$\beta$ regime can be written in the form [11, 12, 13]:

$$\frac{\omega^2}{\omega_A^2}\nabla_\perp^2\varphi + q^2R^2\nabla_\parallel\nabla_\perp^2\nabla_\parallel\varphi - \frac{\omega_{BAE-CAP}^2}{\omega_A^2}\nabla_\perp^2\varphi = 0 \tag{1}$$

where φ is the perturbed scalar potential, $\omega_A = v_A/qR$ is the Alfvén frequency, q the safety factor, R the torus major radius and $\omega_{BAE-CAP}$ the BAE-CAP frequency. Here, $\nabla_\parallel$ and $\nabla_\perp$ are the gradients calculated along and perpendicularly to the total magnetic field, that is the sum of the equilibrium and the island magnetic fields. We introduce the coordinate system (q,u,ζ), with the safety factor $q = (rB_{tor})/(RB_{pol}) \equiv \varepsilon B_{tor}/B_{pol}$ as radial coordinate, given B_{tor} constant and B_{pol} a function of the minor radius r, ζ the direction of the original sheared equilibrium magnetic field (without magnetic islands) at the rational surface $q = q_0$, and u the helical variable, perpendicular to both q and ζ. Since we are only interested in the SAW continuous spectrum modified by the finite size magnetic island at the rational surface $q = q_0$, the modes coupled by the presence of the island are the modes with its same helicity, namely $m = q_0 n$. We define the magnetic flux function $\psi = \Delta q^2/2 + M(\cos(u) + 1)$, with $M = (q_0|s|)(B_{isl}/B_{pol})$, s being the magnetic shear. The ψ value is zero at the *O-point* of the island and $\psi_{sx} = 2M$ at the separatrix. Using this system of coordinates, the nonlinear SAW continuous spectrum is defined as the set of geometric loci in the frequency space for which the solution φ is singular in ψ. By separating the two-scale behavior of the perturbation, the problem reduces to solving an ordinary differential equation along the field lines, admitting the solution $\omega^2 = \omega_{SAW}^2(\psi)$.

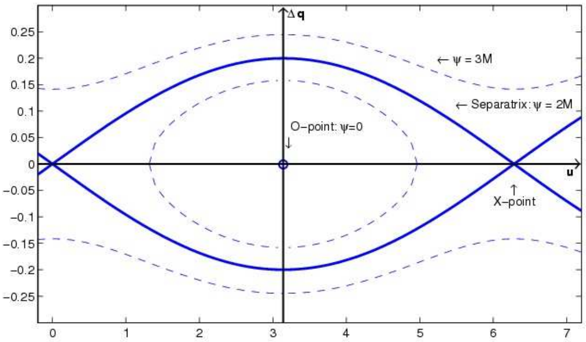

FIG. 1.: Island coordinate system. The horizontal axis $\Delta q = 0$ is the rational surface of the island. In this example the amplitude of the island is chosen as $M = 10^{-2}$.

We solved this equation numerically, with a shooting method both inside and outside the separatrix, and compared the solutions with those obtained analytically far from the island and near the *O-point*. The BAE-CAP is found to be shifted in space from the rational surface of the island ($q = q_0$) to the separatrix flux surface position (labeled $\psi = \psi_{sx}$ in FIG. 2.), following the behavior of the magnetic field helicity pointed out in [14, 15, 16]. The BAE-CAP frequency, $f_{BAE-CAP}$ (we use the $f = \omega/2\pi$ notation for frequency), is not modified by the presence of the island. Outside the island, the branches of the continuous spectrum relative to the various n modes asymptotically reach the $f_\infty = (f_{BAE-CAP}^2 + f_A^2(2\psi - 2M))^{1/2}$ behavior, which physically corresponds to the usual SAW continuum in the absence of the is-

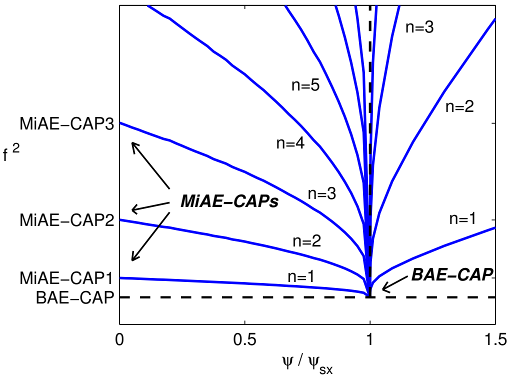

FIG. 2.: Continuous spectrum frequency for various n-modes. The BAE continuum accumulation point is shown at the separatrix with the new MiAE continuum accumulation points at the O-point.

land. The nonuniformity of the magnetic field intensity along the field lines generates a splitting between the frequency branches of the modes with even and odd structures. The most remarkable novel feature is the presence of new continuum accumulation points at the *O-point* of the island ($\psi = 0$), which depend on the toroidal mode number n, and, thereby, give rise to gaps in the continuous spectrum and regions free of continuum damping. This fact could make the existence of new magnetic-island induced Alfvén Eigenmodes (MiAE) possible, excited via wave-particle resonances, provided that the island size is sufficiently wide with respect to the mode radial localization. The MiAE-CAP frequencies are given by:

$$f_{MiAE-CAP} = f_{BAE-CAP}\sqrt{1 + q_0|s|\frac{B_{isl}}{B_{pol}}\frac{f_A^2}{f_{BAE-CAP}^2}l(l+2)} \ , \qquad (2)$$

with $l = 1, 2, \ldots.$ Here q_0, s, and B_{pol} are respectively the values of the safety factor, shear and poloidal magnetic field calculated at the rational surface of the island, B_{isl} is the amplitude of the island radial magnetic field. For small magnetic islands, the scaling is linear with the amplitude and the approximate value is:

$$f_{MiAE-CAP} \simeq f_{BAE-CAP} + \frac{q_0|s|}{2}\frac{B_{isl}}{B_{pol}}\frac{f_A^2}{f_{BAE-CAP}}l(l+2) \ . \qquad (3)$$

The regime of validity of the linear approximation is given by:

$$\frac{B_{isl}}{B_{pol}}l(l+2) \ll \frac{1}{q_0 s}\frac{f_{BAE-CAP}^2}{f_A^2} \sim \frac{\beta}{q_0 s} \ ,$$

which can be broken for high mode numbers ($l(l+2) > \beta/M$) even for finite-β plasmas. In the case of low-β plasmas, it is worthwhile noting that the order of magnitude of the island-induced frequency shift can be comparable with the BAE-CAP frequency itself.

3. Experimental observations

FIG. *3.* shows a spectrogram from Mirnov coils for FTU shot #25877. The lower lines ($0 \leq f_{\text{exp}} \leq 10$ kHz) represent an $(m, n) = (-2, -1)$ magnetic island that forms, then locks and unlocks several times. The lines at higher frequencies (30 kHz $\leq f_{\text{exp}} \leq 65$ kHz) have been interpreted as BAE in a previous work [20] based on linear kinetic analysis neglecting the magnetic island induced frequency shift. In the interval 0.20 s $\leq t \leq 0.25$ s f_{exp} increases, the same

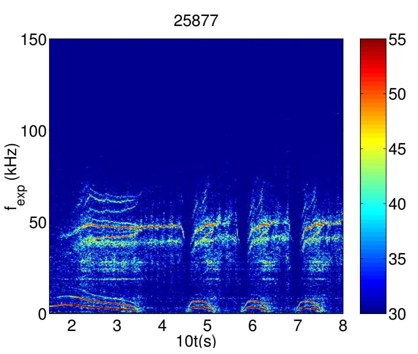

FIG. 3.: Spectrogram (i.e. spectral amplitude versus time t and frequency f_{exp}) of the signal from poloidal field Mirnov coils in FTU shot #25877.

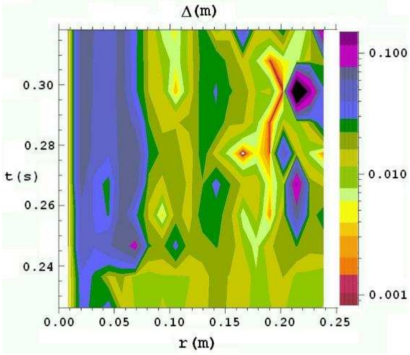

FIG. 4.: Soft X-Ray tomography, showing the plasma displacement Δ versus time and radial position. At $q = 2$ ($r = 14$cm), the displacement $\Delta \simeq 3$cm is the magnetic island width.

phenomenology appearing after each mode unlocking. The magnetic island width can be measured with a soft X-Ray diagnostic. FIG. *4.* shows a soft X-ray tomographic reconstruction of the plasma displacement Δ measured at various radial positions and times in FTU shot $\#25877$. The displacement at $q = 2$ ($r = 14cm$) corresponds to a saturated magnetic island. The island width oscillates in time around a value of $\Delta \simeq 3cm$.

Looking closely at the first $f_{\exp}$ increase in FIG. *3.*, it is possible to plot the frequency versus the island magnetic field fluctuation for the strongest mode, i.e. the one with $f_{\exp} \sim 45$ kHz. The actual mode frequency is approximately the average of the two branches, since the observed frequency splitting of the dominant modes is due to the Doppler shift and vanishes when the island locks [16, 20]. FIG. *5.* shows $f_{\exp}$ versus B_{isl}/B_{pol}, from which a linear scaling is clearly seen for small values of the island width. In this plot the value of B_{isl}/B_{pol} is obtained by appropriate rescaling of Mirnov coil measurements, consistently with the direct measurement of the island width from soft X-Rays tomographic data presented in FIG. *4.*. A picture similar to FIG. *5.* was shown in [16], but there only the lower dominant BAE branch was taken into account, explaining why there is a different scaling. Another difference of the present plot with respect to that shown in [16] is that $\delta B_{pol}(a)/B_{pol}(a)$ were reported there on the abscissas, where δB_{pol} is the poloidal component of the island fluctuating field. In section 4. we compare the experimental data shown in FIG. *5.* with the theoretical MiAE-CAP frequency given by Eq. (3). Most of the quantities can be taken directly from the experimental observations, but ion temperature T_i and magnetic shear s have to be calculated by predictive simulations with the JETTO code [23].

4. Discussion and interpretation of the experimental observations

In section 3., we have presented data relative to experimental observations in FTU plasma shot $\#25877$. These observations were made in the presence of an $(m, n) = (-2, -1)$ magnetic island. The frequencies have a clear dependence on the magnetic island size, starting from $f \simeq 35$ kHz for vanishing island and reaching $f \simeq 45$ kHz when the island is saturated at $B_{isl}/B_{pol} \simeq 5 \cdot 10^{-3}$ (corresponding to $\Delta \simeq 3cm$ if we take $s = 1$). This scaling is consistent with Eq. (2), where the MiAE-CAP theoretical frequency is given as a function of B_{isl}/B_{pol}. In fact, during the island growth, the observed mode frequency increases linearly as well, at least in the early phase, when the island is small (see Eq. (3)). The fact that observed mode frequencies fall below those of the BAE-CAP, $f_{BAE-CAP} \simeq 60$ kHz ($T_e = T_i = 0.5$ keV, $R = 93.5$ cm, $f_A = 1.2 \cdot 10^3$ kHz), confirms the interpretation of these fluctuations as BAE nonlinearly interacting with the magnetic island [20], as indicated in FIG. *6.*. On the contrary of the BAE-CAP frequency, the BAE frequency depends on the magnetic island size. The study of frequency and mode structure of both MiAE and BAE in the presence of a magnetic island is still in progress and will be reported in a separate work.

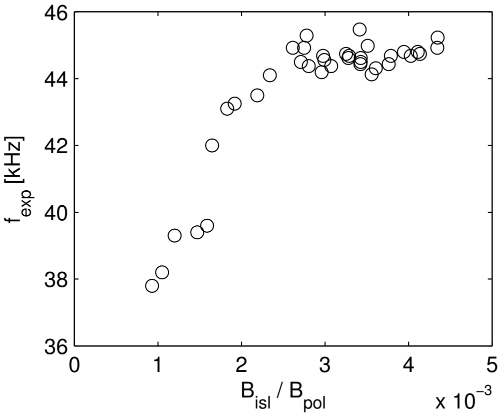

FIG. 5.: $f_{\exp}$ versus B_{isl}/B_{pol} as obtained by appropriate rescaling of Mirnov coil measurements, for the first $f_{\exp}$ increase displayed in FIG. 3.

In section 2., we have shown that the SAW continuous spectrum of a finite-beta tokamak plasma in the presence of a magnetic island still presents the beta-induced gap at low-frequencies and that the gap width is not changed by the presence of the island. This window in the Alfvén continuum, namely $0 < f < f_{BAE-CAP}$, allows BAE to grow free of continuum damping in the proximity of the BAE continuum accumulation point [11, 12, 13, 24, 25]. We have also shown that new local structures of the SAW continuum are formed due to the presence of the magnetic island and that new continuum accumulation points (MiAE-CAPs) arise at higher frequencies. The frequency range $f_{BAE-CAP} < f < f_{MiAE-CAP}$ is the window where new eigenmodes could grow in the presence of a suitable driving free energy source. The substantial difference between these two frequency windows is given by the radial continuum structure. In fact, a mode with frequency inside the beta-induced gap can extend radially outside the separatrix without resonantly exciting the SAW continuum. On the other hand, the magnetic-island induced frequency window is defined as the window between the two accumulation points. This implies that a localized plasma eigenmode can exist inside the magnetic-island induced gap only if its fluctuating field vanishes sufficiently fast (exponentially) at the continuum position (see FIG. *6.*). If the fluctuating field is finite at the continuum position, its radial structure is characterized by the logarithmic singularity typical of continuum resonance and absorption and could be excited only as EPM above a critical drive threshold [4]. This argument gives us precise information on the radial structure of the new MiAE with respect to gap-modes like BAE. In fact, we can state that, if a BAE can extend over a radial range not limited by the island size, on the contrary a MiAE is very localized and positioned at the center of the island. As a consequence, a MiAE cannot be easily detected by a diagnostic system which measures the fluctuating field outside the tokamak, such as Mirnov coils. However, we expect that we could observe these modes with ECE or soft X-ray diagnostics.

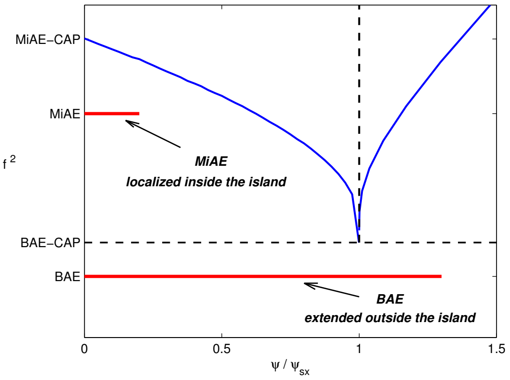

FIG. 6.: MiAE and BAE relative frequency and localization. The MiAE has a frequency higher than the BAE-CAP and is localized at the center of the island. The BAE has a frequency below the BAE-CAP and is more extended in space.

5. Conclusions

In this work, the continuous spectrum of shear Alfvén waves has been calculated for finite-β tokamak equilibrium in the presence of a finite-size magnetic island. The beta-induced Alfvén Eigenmodes continuum accumulation point (BAE-CAP) is found to be shifted in space from the rational surface of the island to the separatrix flux surface position, while the frequency f_{BAE} remains the same. New continuum accumulation points (MiAE-CAPs) are found at the *O-point* of the island and, consequently, new gaps in the continuous spectrum. This result has potential implications in analysis of stability properties of tokamak plasmas in presence of magnetic islands. In fact, new magnetic-island induced Alfvén Eigenmodes (MiAE) could be excited inside a magnetic island if the thermal or energetic component of the plasma provide sufficient free energy for driving the mode. We have discussed the relative radial localization of MiAE, placed in the center of the island, with respect to the BAE, which are generally extended outside the island separatrix. Unlike in the BAE case, the radial MiAE localization at the center of the island makes them difficult to be detected by diagnostic systems, which measure the fluctuating fields at the plasma boundary, such as Mirnov coil systems. We expect to possibly detect MiAE with diagnostics such as ECE and soft X-Rays.

We have presented data relative to FTU experimental observations of Alfvénic modes in the presence of a magnetic island and shown their frequency scaling with the island amplitude. The fluctuations frequency range confirms that they are BAE nonlinearly interacting with the magnetic island, characterized by a frequency shift dependence on the island size analogous to that of the MiAE-CAP scaling. The study of frequency and structure of both MiAE and BAE in the presence of a magnetic island is in progress and will be reported in future works along with the kinetic study of the respecting driving mechanisms. Here, we want to emphasize that our present theoretical approach is not limited to finite magnetic shear. For $s = 0$, i.e. the typical condition under which reversed shear Alfvén Eigenmodes can be excited [17, 18], we can easily generalize Eq. (2) by simply considering that the BAE-CAP is shifted by $k_{\|,0}^2 v_{A,0}^2$, with $k_{\|,0}$ and $v_{A,0}$ the parallel wave vector and the Alfvén speed at the minimum-q surface [26, 27]. Due to the dependence of the MiAE-CAP frequency on mode numbers and the magnetic island size, the possibility of using Eqs. (2) and (3) as novel magnetic island diagnostic is an attractive option.

6. *Acknowledgments

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