

Criteria for Runaway Electron Generation in Tokamak Disruptions

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Abstract. Experimental observations show that there is a magnetic field threshold for runaway electron generation in tokamak disruptions. A possible explanation for these observations is that the runaway beam excites whistler waves that scatter the electrons in velocity space and prevents the beam from growing. The growth rates of the most unstable whistler waves are inversely proportional to the magnetic field strength. Taking into account the collisional and convective damping of the waves it is possible to derive a magnetic field threshold below which no runaways are expected. In this work, this magnetic field threshold is compared with a criterion for substantial runaway production obtained by calculating how many runaway electrons can be produced before the induced toroidal electric field diffuses out of the plasma. It is shown, that even in rapidly cooling plasmas, where hot-tail generation is expected to give rise to substantial runaway population, the whistler waves can stop the runaway formation below a certain magnetic field unless the post-disruption temperature is very low.

1. Introduction

Due to a sudden cooling of the plasma in tokamak disruptions a beam of relativistic runaway electrons is sometimes generated, which can cause damage on plasma facing components due to highly localized energy deposition. This problem becomes more serious in larger tokamaks with higher plasma currents and understanding of the processes that may limit or eliminate runaway electron generation is very important for future tokamaks, such as ITER. In present tokamak experiments it is observed that the number of runaway electrons generated depends on the magnetic field strength. Several large tokamaks have reported that no runaway generation occurs unless the magnetic field B exceeds 2 T [1, 2]. Above this threshold, the runaway generation shows a non-linear dependence on B , and a doubling of B results in an increase of the photo-neutron production by two orders of magnitude [3]. The whistler wave instability (WWI) excited by runaway electrons may provide an explanation for this observation. The growth rates of the most unstable whistler waves are inversely proportional to the magnetic field strength and the WWI causes a rapid pitch-angle scattering of the runaways and may stop runaway beam formation [4, 5]. The aim of the present work is to analyze the magnetic field threshold for runaway generation due to the WWI and compare with the criterion for runaway avalanche (CRASH) based on the coupled dynamics of plasma current and runaway generation. This theoretical criterion for substantial runaway generation has been derived in [6, 7], but without the effect of the hot-tail generation of runaways [8]. In this work we derive more complete criteria both for the WWI by taking into account the localization of the runaway beam that gives rise to convective damping of the waves and for CRASH by including the hot-tail generation of runaways. We analyze the relevance

of the two different mechanisms for JET and ITER-like disruption scenarios. It is shown, that even in rapidly cooling plasmas, where hot-tail generation is expected to give rise to substantial runaway population, the whistler waves can stop the runaway formation at a certain magnetic field (of the order of 2 T in JET) unless the post-disruption temperature is very low (below 10 eV).

2. WWI

The runaway electron beam has a strongly anisotropic velocity distribution. When the degree of anisotropy exceeds a critical level, unstable whistler waves, with frequencies well below the non-relativistic electron cyclotron frequency ω_{ce} but above the ion cyclotron frequency ω_{ci} are excited. Assuming $k_{\perp}^2 v_{Te}^2 \ll \omega^2$, where v_{Te} is the electron thermal velocity, the dispersion relation of these waves can be simplified to [5]

$$k^2 v_A^2 \left(1 + \frac{k_{\parallel}^2 v_A^2}{\omega_{ci}^2} + \frac{k_{\perp}^2}{k^2} \right) - \omega^2 \left(1 + \frac{(k^2 + k_{\parallel}^2) v_A^2}{\omega_{ci} \omega_{ce}} \right) = \omega^2 \frac{\omega_{ci}^2}{\omega_{pi}^2} \left[\left(1 + \frac{k^2 v_A^2}{\omega_{ci}^2} \right) \chi_{11}^r + \left(1 + \frac{k_{\parallel}^2 v_A^2}{\omega_{ci}^2} \right) \chi_{22}^r + 2i \frac{\omega}{\omega_{ci}} \chi_{12}^r \right], \quad (1)$$

where ω is the wave frequency, k_{\parallel} and k_{\perp} are the parallel and perpendicular components of the wave number, $v_A = c\omega_{ci}/\omega_{pi}$ is the Alfvén velocity, ω_{pi} is the ion plasma frequency, c is the speed of light, χ_{ij}^r denotes the runaway contribution to the susceptibility tensor. Without runaways, for $k_{\parallel}^2 c^2/\omega_{pi}^2 \gg 1$ and $(k^2 + k_{\parallel}^2) v_A^2 \ll \omega_{ci} \omega_{ce}$, the dispersion relation can be further simplified to obtain the usual relation for the whistler wave $\omega = k k_{\parallel} v_A^2/\omega_{ci}$.

Numerical simulations in [4] showed that the most important interaction occurs at the anomalous Doppler resonance $\omega - k_{\parallel} v_{\parallel} = -\omega_{ce}/\gamma$, where v_{\parallel} is the particle velocity parallel to the magnetic field and γ is the relativistic factor. The linear growth rates of these waves are such that they are stable for high magnetic field (so the runaway beam can form) but unstable for low magnetic field. In Ref. [4] it was shown that the linear growth rate of a small perturbation $\omega = \omega_0 + \delta\omega$ is $\gamma_i = \text{Im}\delta\omega = (k - k_{\parallel})^2 v_A^2 \omega_0 \text{Im}\chi^r(t)/2\omega_{pi}^2$, with

$$\chi^r(t) = \frac{\omega_{pr}^2 \omega_{ce}^2}{k_{\perp}^2 \omega c^2} \int d^3p \frac{J_1^2(z)}{\omega(\omega\gamma - k_{\parallel} p_{\parallel} c + \omega_{ce})} \left(\frac{\omega - k_{\parallel} p_{\parallel} c/\gamma}{p_{\perp}} \frac{\partial f_r}{\partial p_{\perp}} + \frac{k_{\parallel} c}{\gamma} \frac{\partial f_r}{\partial p_{\parallel}} \right). \quad (2)$$

In this expression, $\omega_{pr}^2 = n_r e^2/m_{e0}\epsilon_0$, and J_n is the Bessel function of the first kind and order n with the argument $z = k_{\perp} p_{\perp} c/\omega_{ce}$. The quantity $f_r = f/n_r$ is the normalized secondary runaway distribution function [4]

$$f(p_{\parallel}, p_{\perp}, t) = \frac{\alpha n_r}{2\pi c_Z p_{\parallel}} \exp\left(-\frac{p_{\parallel}}{c_Z} - \frac{\alpha p_{\perp}^2}{2p_{\parallel}}\right), \quad (3)$$

where $\alpha = (\hat{E} - 1)/(1 + Z)$, $\hat{E} = e|E|\tau/m_{e0}c$ is the normalized parallel electric field, $\tau = 4\pi\epsilon_0^2 m_{e0}^2 c^3/n_e e^4 \ln \Lambda$ is the collision time for relativistic electrons, m_{e0} is the electron rest mass, Z is the effective ion charge and $c_Z = \sqrt{3(Z + 5)}/\pi \ln \Lambda$. The distribution in (3) is valid if $\hat{E} \gg 1$ and secondary generation of runaways is dominant, as expected to be the case in large tokamak disruptions. The runaway density grows as $dn_r/dt = (\hat{E} - 1)n_r/c_Z\tau$ [9], giving $n_r = n_{r0} \exp\left[(\hat{E} - 1)t/(\tau c_Z)\right]$ if the electric field is assumed constant in time,

and where n_{r0} denotes the seed produced by primary (Dreicer+hot-tail) generation.

Usually, in low-temperature plasmas, the dominant damping process is the electron-ion collisional damping, with $\gamma_d \simeq 1.5\tau_{ei}^{-1}$ [10], where $\tau_{ei} = 3\pi^{3/2}m_{e0}^2v_{Te}^3\epsilon_0^2/n_iZ^2e^4 \ln \Lambda$ is the electron-ion collision time. If the runaway beam is localized with an effective beam radius L_r then the wave-convection out from the region where the beam is localized gives a damping term $\gamma_v \equiv (\partial\omega/\partial k_\perp)/(4L_r) = v_A^2k_\parallel/4\omega_{ci}L_r$ [5]. At low density, high temperature and strong magnetic field this term can be comparable to the collisional damping γ_d . The total linear growth rate of the WWI is then $\gamma_l = \gamma_i - \gamma_d - \gamma_v$.

The linear stability analysis in [4] has shown that the frequency, growth rate and wave number of the most unstable wave are approximately $\omega_0 = \omega_{ce}/cZ$, $\gamma_i = 1.3 \cdot 10^{-9}n_r/B_T$, $k_0 = \omega_{pi}/2v_A = 3 \cdot 10^4n_{19}B_T \text{ m}^{-1}$, and $k_{\parallel 0} = 2\omega_{ce}/(cc_Z) = 30B_T \text{ m}^{-1}$, where B_T is the toroidal magnetic field in Teslas and $n_e = n_{19}10^{19} \text{ m}^{-3}$ is the electron density. These values for the most unstable frequency and wave number have been calculated without taking into account the convective damping, the effect of which can be shown to reduce the values of k_0 and $k_{\parallel 0}$ (still within the assumptions applied here) but the growth rate γ_i is quite insensitive to the exact magnitude of k_0 and $k_{\parallel 0}$ [4].

From the linear instability threshold $\gamma_l > 0$ for the most unstable wave one can derive a threshold for the fraction of runaways required for destabilization

$$\frac{n_r}{n_e} > \frac{Z^2B_T}{20T_{eV}^{3/2}} + \frac{B_T^3}{90c_Zn_{19}^2L_r} \quad (4)$$

where T_{eV} is the post-disruption electron temperature in eVs. Here, the first term on the right hand side is due to the collisional damping and the second term is due to the convective damping of the WWI. The lower the magnetic field and higher the post-disruption temperature the less runaways are needed for the destabilization of WWI. The threshold presented in (4) is calculated using the wave numbers k_0 and $k_{\parallel 0}$ given above. Numerical simulations show that the most unstable wave numbers are lower if convective damping is taken into account and slightly less runaway electrons are needed for destabilization [5].

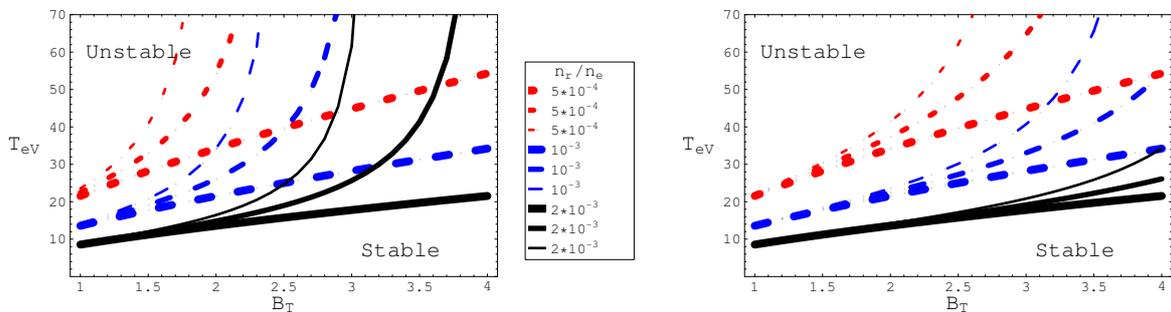


Figure 1: Stability threshold from Eq.(4) for different L_r . The thickest line is for $L_r = \infty$, the line with middle-thickness is for $L_r = 0.4 \text{ m}$ and the thinnest line is for $L_r = 0.2 \text{ m}$. Left figure is for $n_e = 5 \cdot 10^{19} \text{ m}^{-3}$ and right figure is for $n_e = 10^{20} \text{ m}^{-3}$.

Figure 1 shows the stability threshold from Eq.(4) for different beam radii and fractions of runaways. The thickest line represents the case with a wide runaway beam so that

convective damping is negligible. The thinner lines correspond to thinner runaway beams. Interestingly, the decreasing runaway beam radii lead to lower magnetic field thresholds for a given post-disruption temperature and these values for the magnetic field threshold become effectively independent of the temperature above a certain value. Note specifically in the left figure, that the curve corresponding to the parameters $L_r = 0.2$ m and $n_e = 5 \cdot 10^{19} \text{ m}^{-3}$ shows a threshold in the magnetic field around 2 T for $n_r/n_e = 10^{-3}$.

The evolution of the runaway distribution and the whistler wave spectral energy coupled through the quasi-linear diffusion process is illustrated on Figure 2 based on the study [5]. As a result of the wave-particle interaction, the particle distribution becomes more isotropic. As the main driving term $\partial f/\partial p_\perp$ becomes smaller, the whistler wave becomes marginally stable ($\gamma_k \lesssim 0$), but as the runaway density increases due to collisional processes, the wave is destabilized again, which leads to further velocity isotropization. If the plasma parameters are such that the whistler wave is destabilized, the time-scale of the isotropization is of the order of 10^{-5} s. Note that the time-scale of the linear and quasi-linear evolution of the instability is much shorter than the runaway avalanche growth time. The pitch-angle diffusion leads to higher synchrotron radiation emission [11] that lowers the runaway electron energy and this should lead to a rapid quench of the resonant part of the runaway beam as soon as the runaway density reaches the threshold.

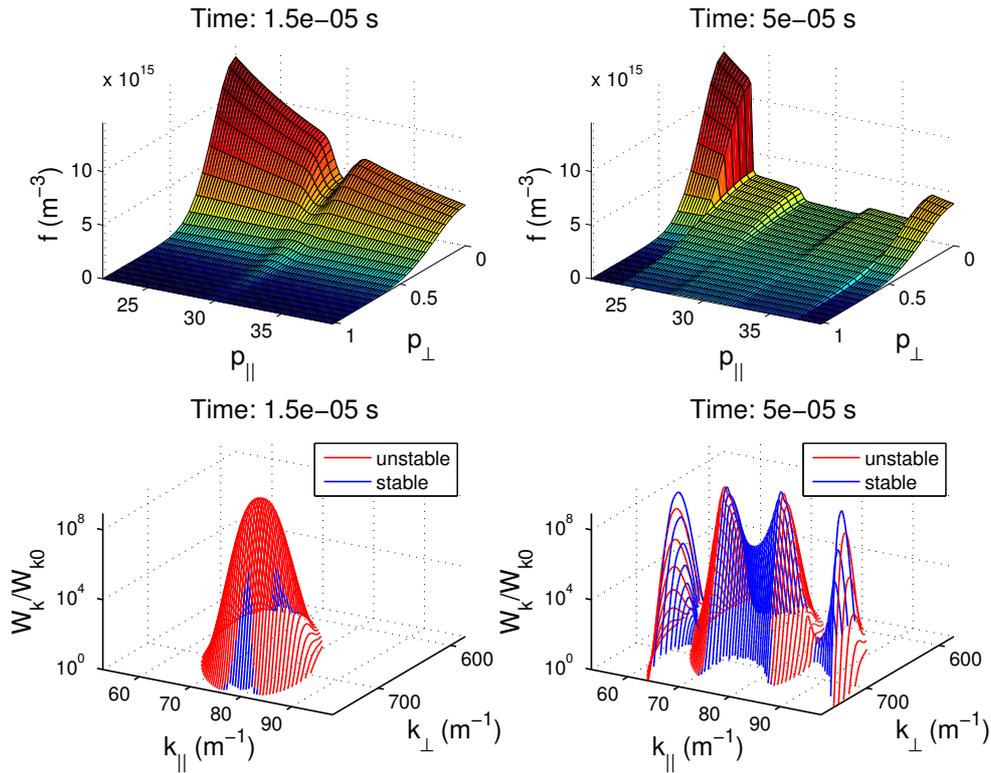


Figure 2: Runaway electron distribution functions and wave spectral energies plotted for two time slices of the quasi-linear interaction of whistler waves and runaway electrons: beginning of the quasi-linear diffusion and development of the second phase ($B = 2$ T, $n_e = 5 \cdot 10^{19} \text{ m}^{-3}$, $T_e = 10$ eV, $E_{||} = 40$ V/m, $L_r = 0.2$ m)

3. CRASH

The magnetic field dependence of runaway generation is introduced via the on-axis current density $j_0 = 2B/\mu_0qR$, which is proportional to the magnetic field if the central safety factor q can be assumed to be limited to a value around 1–2 due to operational constraints. A higher on-axis current density leads to higher post-thermal quench electric field, which leads to stronger initial runaway generation by the Dreicer and hot-tail mechanisms. The “seed” produced by these processes is amplified by the secondary avalanche mechanism, the strength of which depends on the total plasma current I_0 . The runaway population in tokamaks with large current (e.g. JET, ITER) is mainly produced by the avalanche, but it is very sensitive to the seed runaway density, and therefore also to the magnetic field.

Based on the approximate solution of two coupled differential equations for the runaway electron density and plasma current, a criterion for substantial runaway generation was first derived in [6] and later refined in [7]. A zero-dimensional model describes the time-dependence of the electric field E induced by the falling current I

$$E \approx -\frac{L}{2\pi R} \frac{dI}{dt} \quad (5)$$

where the plasma inductance can be assumed to be $L \simeq \mu_0 R$. The current is the sum of the Ohmic and runaway currents $I = jA_{\text{eff}} = (\sigma E + n_r ec)A_{\text{eff}}$, where j is the on-axis current density, A_{eff} is an effective cross section area of the current, and σ is the conductivity. The criterion derived in [6] and [7], which included only Dreicer and avalanche runaway generation, will here be extended to account also for the hot tail mechanism.

Recent work [8] derived analytical estimates for the amount of hot-tail runaway electrons generated in plasmas with an exponential temperature decrease given by $T = T_0 e^{-t/t_0}$. For a sudden temperature decrease, the hot-tail generated runaways are given by

$$n_h = n_0 \frac{2}{\sqrt{\pi}} u_c e^{-u_c^2}, \quad (6)$$

where

$$u_c^3 = t_0 \nu_0 \left[2 \ln \frac{E_{D0}}{2E_0} - \frac{4}{3} \ln \left(\frac{4}{3} t_0 \nu_0 \right) - \frac{5}{3} \right]. \quad (7)$$

In these expressions, $\nu_0 = n_0 e^4 \ln \Lambda / (4\pi \epsilon^2 m_e^2 v_{T0}^3)$ is the initial (pre-disruption) collision frequency of the thermal electrons, n_0 and v_{T0} are the initial background electron density and thermal speed, and E_{D0}/E_0 is the initial ratio of the Dreicer field and parallel electric field.

We assume that the density of seed runaways is the sum of the hot-tail runaway density and the Dreicer generated runaway density. If the runaway current remains a small fraction of the total current, the seed runaway population will be amplified a factor e^α by the avalanche, and the total number of runaways will be $n_r = (n_D + n_h)e^\alpha$, where $\alpha = (\sqrt{2\pi}/3)(I_0/I_A \ln \Lambda)$ and $I_A = 0.017$ MA is the Alfvén current. On the other hand, if the runaway current replaces a large fraction of the Ohmic current, this will cause the electric field to decrease, and the runaway current to saturate before it reaches the initial current I_0 . Since this saturation mechanism only comes into play when a considerable runaway current fraction has already been produced, it is not necessary to take it into

account in order just to estimate whether or not this will happen. For this, one only needs to determine if $S \equiv \ln n_r > 0$, so the criterion for large runaway production is

$$S = \alpha + \ln \left[\frac{\sqrt{2}\alpha \ln \Lambda}{\pi} \frac{m_e c^2}{T_e} \left(\frac{E_D}{E} \right)^{\frac{11}{8}} e^{-\frac{E_D}{4E} - \sqrt{\frac{2E_D}{E}}} + \frac{E_D}{E} \sqrt{\frac{m_e c^2}{T_e}} \frac{\sqrt{2}u_c}{3\pi} e^{-u_c^2} \right] > 0, \quad (8)$$

where E/E_D and T_e should be evaluated after the thermal quench. Due to the weak dependence of the hot tail generation on E_D/E compared with the dependence on $\nu_0 t_0$, the B_T dependence of S comes mainly from the Dreicer mechanism. The avalanche term α depends only on I_0 , which we choose to vary independently of B_T .

The runaway current evolution was calculated using a zero-dimensional numerical model based on equation (5) and the equation for runaway generation including hot-tail generation according to the velocity moment method described in Ref. [8]. This shows that the CRASH criterion gives a good approximation for the parameter region of significant runaway production.

4. Discussion

Similarly to the WWI-threshold, CRASH shows that runaway generation is expected only for magnetic fields above a certain threshold, which depends on the initial plasma current and the electron density and temperature. Approximately, for JET-like values ($q = 1.5$, $R = 3$, $I_0 = 2$ MA), when hot-tail generation is not very strong, (8) can be written as

$$B_T > n_{19} \sqrt{T_{eV}/I_{MA}}/4, \quad (9)$$

where I_{MA} is the initial plasma current in megaamperes. Note that to keep q and I_0 constant while varying B_T (or equivalently j_0) corresponds to varying the current cross section A_{eff} . For ITER-like values ($q = 1.5$, $R = 6$, $I_0 = 15$ MA), the hot-tail generation is dominant and (8) is always positive, unless $\nu_0 t_0$ is very large. This means that CRASH does not lead to a threshold in the magnetic field for ITER-like parameters.

Figure 3 compares the magnetic field determined by WWI and CRASH for JET-like and ITER-like parameters. Above the CRASH-lines $S > 0$ and substantial runaway generation is expected. Above the WWI-lines whistler waves are stable and a runaway beam can form. For temperatures less than ~ 10 eV, the magnetic field threshold for stability of whistler waves is very low, and therefore the WWI will not stop the runaway beam formation. But for temperatures above 10 – 20 eV, as the convective damping becomes comparable to collisional damping, the temperature dependence becomes less important and the WWI leads to a threshold around 2 T for both JET-like and ITER-like parameters. Due to the strong magnetic field dependence of the convective damping, the threshold for WWI turns out to be only weakly dependent of the other plasma parameters. A more exact threshold for the destabilization can be obtained by numerical simulations described in [5], and this would shift the WWI-curves toward lower temperatures by about 20-30%.

In the JET-like case CRASH leads to a higher magnetic field threshold than WWI. This means that the zero-dimensional model presented in Sec. 3 does not predict substantial runaway generation below the magnetic field indicated by the solid and dotted lines for the chosen cooling time. Therefore one may draw the conclusion that there is no runaway

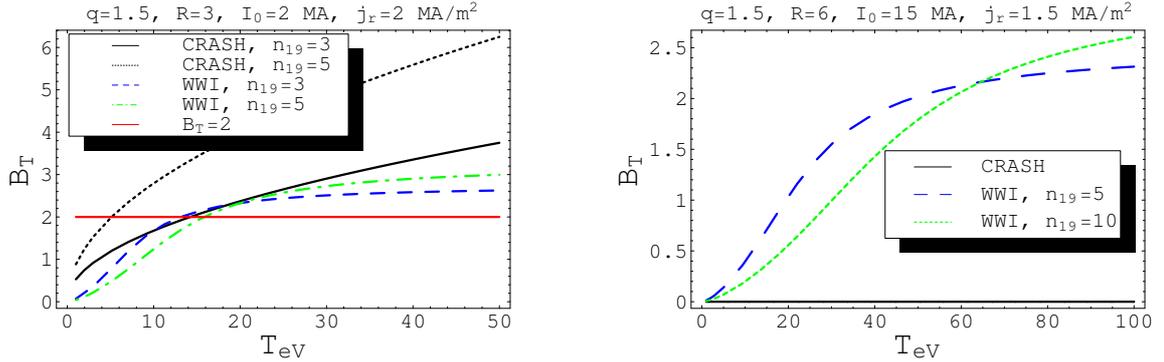


Figure 3: Critical magnetic field for significant runaway generation as a function of T_{eV} for different electron densities. (left) JET-like parameters: $q = 1.5$, $R = 3$, $I_0 = 2$ MA, $\nu_0 t_0 = 10$ (for CRASH), $L_r = 0.2$ m, $j_r = 2$ MA/m² (for WWI). (right) ITER-like parameters: $q = 1.5$, $R = 6$, $I_0 = 15$ MA (for CRASH), $L_r = 0.3$ m, $j_r = 1.5$ MA/m² (for WWI).

beam for the WWI to stop. However, if the cooling time is shorter, CRASH leads to a lower magnetic field threshold, and below $t_0 \simeq 1$ ms (for $q = 1.5$, $R = 3$, $I_0 = 2$ MA, $T_{e0} = 3$ keV, $n_{19} = 3$, $B_T = 2$), S is always positive. Note that CRASH depends approximately linearly on the electron density, and the probability for runaway production is higher for low density. In ITER-like disruptions, the current quench time is predicted to be $t_0 = 1 - 10$ ms [12], so that $\nu_0 t_0 < 5$ and runaways are always likely to be produced due to the hot-tail generation, although the beam formation will probably be stopped by WWI below 2 T.

We have demonstrated that the WWI is an effective loss mechanism for secondary runaways. However, there are several other processes that can limit the runaway energy or cause loss of runaways, such as synchrotron radiation [11], Bremsstrahlung [13], unconfined drift orbit losses [14], resonance between gyro-motion and magnetic field ripple [15], and radial diffusion due to magnetic field fluctuations [16]. Since these mechanisms are not considered in this work, the results presented here are expected to give a lower limit of the magnetic field threshold for runaway production.

5. Conclusions

For a given temperature, density and runaway fraction, if the magnetic field is below a critical value, the whistler wave can be destabilized by relativistic secondary runaway electrons. This mechanism offers a possible explanation for the magnetic field threshold for runaway generation observed in tokamak disruptions. Lower runaway fractions are needed for destabilization in plasmas with high temperature, since then the collisional damping is weaker. The convective damping due to the localization of the runaway beam can be of the same order of magnitude as the collisional damping for high temperature and strong magnetic field. The convective damping is sensitive to the radius of the runaway beam and the fraction of runaways in the plasma n_r/n_e , and these depend on the other plasma parameters, for instance the final temperature and the cooling time. Therefore, runaway production and suppression by WWI is a dynamical process, in which the runaways trigger the WWI. The consequent scattering affects the strength and width of the runaway beam, which in turn affects the damping of the WWI. It is difficult to predict exactly where the

threshold in B might be without self-consistent simulations of the runaway distribution function and electric field evolution, that could be achieved for instance by the ARENA code [17] coupled to an evaluation of the instability growth rate.

The magnetic field dependence of the runaway production has been studied by considering the coupled dynamics of the runaway generation and evolution of plasma current, including the hot-tail generation of runaways. This leads to an analytical criterion for runaway avalanche (CRASH) that can be used to estimate if there will be a substantial runaway generation or not. CRASH can be shown to lead to a magnetic field threshold unless hot-tail generation dominates. However, in rapidly cooling plasmas, where hot-tail generation gives rise to a substantial runaway population, the whistler waves can stop the runaway formation at a certain magnetic field unless the post-disruption temperature is too low. If the post-disruption temperature is very low then whistler waves are stable and the runaway beam develops unless its growth is limited by physical processes not taken into account in this paper.

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