Influence of Anisotropy on Radiation of Any Linear Antenna System in Magnetoplasma

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1. Introduction

The studies of linear antenna in anisotropic mediums are important in such applications as radio communication in space, heating of plasma by high-frequency electromagnetic fields, diagnostics of magnetoactive plasma, electromagnetic compatibility of electronic devices, etc. The purpose of the present work is the solution of the problem on excitation of a system of thin impedance antennas arbitrary located in anisotropic plasma and to determine conditions of effective transfer of energy from antenna to plasma.

2. Statement of the problem

We consider a system of antennas located in magnetoactive plasma, permittivity of which is a diagonal tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix},$$

with components $\varepsilon_1 = 1 - \omega_N^2 / \left(\omega^2 - \omega_B^2\right)$, $\varepsilon_3 = 1 - \omega_N^2 / \omega^2$, ω_N is the Lengmur's frequency; ω_B is the Larmor's frequency; ω is the working frequency. Components of dielectric permittivity tensor $\hat{\varepsilon}$ can accept both positive and negative values, and generally they are complex functions of frequency. The magnetic field is directed along axis OZ. In plasma there is the antenna system consisting of two thin linear impedance antennas, any way oriented concerning a direction of an external magnetic field. Thus antennas are not neither parallel, nor perpendicular among themselves. $2L_m$ is the length of the m-th antenna, b_m is its radius, $Z_m = R_m + iX_m$ is the complex skin impedance m-th antennas normalized on 120 π, $m = 1, 2$. The position of antenna system in plasma is shown in Fig.1.

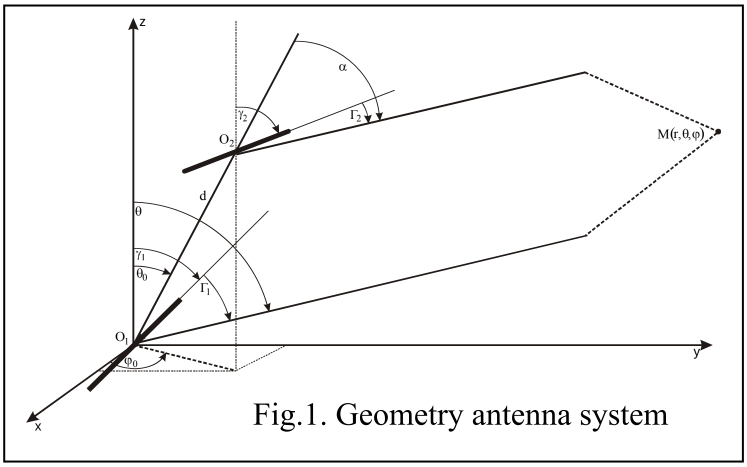

Fig.1. Geometry antenna system

It is necessary to find the current distribution along each antenna, the electromagnetic field of radiation, the condition of effective transfer of energy from antenna system to plasma.

3. Method of the solution.

The solution of this problem is to be obtained on the basis of integral equations of electromagnetics derived from exact expressions of the Green's function for uniaxial anisotropic medium [1]. These equations are completely equivalent of the Maxwell equations and boundary conditions; they can be written as

$$i\omega\sqrt{\varepsilon_1}\left[\vec{E}(\vec{r}_n)-\vec{E}_0(\vec{r}_n)\right]=\left(graddiv+k_0^2\varepsilon_1\varepsilon_3\hat{\varepsilon}^{-1}\right)\left[\vec{A}_{n1}(\vec{r}_n)+A_{n2}(\vec{r}_n)\right]+ik_0rot\vec{e}_z\left[B_{n1}(\vec{r}_n)+B_{n2}(\vec{r}_n)\right], \quad (1)$$

$$i\omega\sqrt{\varepsilon_1}\left[\vec{H}(\vec{r}_n)-\vec{H}_0(\vec{r}_n)\right]=ik_0\varepsilon_1\varepsilon_3\hat{\varepsilon}^{-1}rot\left[\vec{A}_{n1}(\vec{r}_n)+\vec{A}_{n2}(\vec{r}_n)\right]-\left(grad\frac{\partial}{\partial z}-\vec{e}_z\Delta\right)\left[B_{n1}(\vec{r}_n)+B_{n2}(\vec{r}_n)\right], \quad (2)$$

where

$$\vec{A}_{nm}(\vec{r}_n)=\int\limits_{V_m}\frac{\vec{j}_m(\vec{r}_m')\exp\left(-ik_0\sqrt{\varepsilon_3|\vec{\rho}_n-\vec{\rho}_m'|^2+\varepsilon_1(z_n-z_m')^2}\right)}{\sqrt{\varepsilon_3|\vec{\rho}_n-\vec{\rho}_m'|^2+\varepsilon_1(z_n-z_m')^2}}d\vec{r}_m',$$

$$B_{nm}(\vec{r}_n)=\int\limits_{V_m}\left[\vec{j}_m(\vec{r}_m'),grad\,|\vec{\rho}_n-\vec{\rho}_m'|\right]_z\frac{\exp\left(-ik_0\sqrt{\varepsilon_1}|\vec{r}_n-\vec{r}_m'|\right)-\exp\left(-ik_0\sqrt{\varepsilon_3|\vec{\rho}_n-\vec{\rho}_m'|^2+\varepsilon_1(z_n-z_m')^2}\right)}{|\vec{\rho}_n-\vec{\rho}_m'|}d\vec{r}_m',$$

where $\vec{E}_0$, $\vec{H}_0$ - electrical and magnetic fields of a source; $\vec{E}$, $\vec{H}$ - scattering electrical and magnetic fields; $k_0=\omega/c$ - wave number in vacuum, c - velocity of light; V_m - volume of m-th antenna; $\vec{e}_z$ - ort along an external magnetic field (axis of anisotropy); $\vec{r}_n=\vec{r}_n(x_n,y_n,z_n)$ and $\vec{r}_m'=\vec{r}_m'(x_m',y_m',z_m')$ (radius - vectors of points of observation and integration accordingly); the index z in expressions for $B_{nm}(\vec{r}_n)$ points out a projection of vector product on axis of anisotropy; $|\vec{\rho}_n-\vec{\rho}_m'|=\sqrt{(x_n-x_m')^2+(y_n-y_m')^2}$; $\vec{j}_m(\vec{r}_m)$ - volume density of current in m-th antenna; in all formulas $n, m = 1, 2$.

The decision of our problem is received by analogy to more simple problems solved by us earlier, namely, problems about excitation of one antenna [2], and also systems of the parallel [3] and perpendicular [4] antennas located in anisotropic plasma.

The advantage of the method of integral equations [5] is that it enables us, in an analytical form, to solve a large class of boundary problems of electromagnetism. The algorithm of solution by this method contains two stages. At the first stage we find the currents excited in each antenna by the field of the source and the field produced by the other antenna. In this case initial integral equations (1), (2) are system of the non-uniform integral Fredholm's equations of the first kind having the unique solution. At the second stage, based on already known currents we find the total field. In these case equations (1), (2) are simply equality representing a complete field as the sum of a field of a source and a scattering field.

A problem about excitement of electrical current in the antenna system will consider on the base of these integral equations. As far as antennas are thin: $b_i/L_i \langle\langle 1$; $b_i/\lambda \langle\langle 1$, $i=1,2$, where λ is a wavelength of falling electromagnetic field, only that electrical currents will be essential which current along antenna; it is possible to neglect of transverse currents.

Considering only tangential components of full electrical field on surfaces of antenna, we obtain equations for currents, induced in each antenna by given falling field and field, which generate other antenna. For that it's needs introduce two new coordinate systems χ,η,ξ and u,v,w. Let's first antenna is directed along axis $o\eta$ and second antenna is directed along axis ov. Relationship between coordinates of new and old systems is installed by formulas

$$\chi = x,\quad \eta = y\sin\gamma_1 + z\cos\gamma_1,\quad \xi = -y\cos\gamma_1 + z\sin\gamma_1,$$

$$u = x\sin\varphi_2 - y\cos\varphi_2,\ v = x\sin\gamma_2\cos\varphi_2 + y\sin\gamma_2\sin\varphi_2 + z\cos\gamma_2,$$

$$w = -x\cos\gamma_2\cos\varphi_2 - y\cos\gamma_2\sin\varphi_2 + z\sin\gamma_2,$$

where γ_m, φ_m - angular coordinates of the *m*-th antenna, $m = 1,2$ ($\varphi_1 = 90^0$).
Then equations for currents $I_1(\eta_1)$, $I_2(v_2)$ are obtained in the manner of

$$\frac{d^2 I_1(\eta_1)}{d\eta_1^2} + k_0^2\varepsilon_{eq1} I_1(\eta_1) = \alpha_1\sqrt{\beta_1}\Big\{i\omega\sqrt{\varepsilon_1}\left[E_{0\eta 1}(\eta_1) - E_{\eta 1}(\eta_1)\right] -$$

$$-\frac{dI_1(\eta_1')}{d\eta_1'}\frac{\exp(-ik_0R_{11})}{R_{11}}\Bigg|_{\eta_1'=-L_1}^{\eta_1'=L_1} - \cos\Gamma\frac{dI_2(v_2')}{dv_2'}\frac{\exp(-ik_0R_{12})}{R_{12}}\Bigg|_{v_2'=v_0-L_2}^{v_2'=v_0+L_2} +$$

$$+k_0^2\Psi_1\int_{v_0-L_2}^{v_0+L_2} I_2(v_2')\frac{\exp(-ik_0R_{12})}{R_{12}}dv_2'\Big\}, \tag{3}$$

$$\frac{d^2 I_2(v_2)}{dv_2^2} + k_0^2\varepsilon_{eq2} I_2(v_2) = \alpha_2\sqrt{\beta_2}\Big\{i\omega\sqrt{\varepsilon_1}\left[E_{0v2}(v_2) - E_{v2}(v_2)\right] -$$

$$-\frac{dI_2(v_2')}{dv_2'}\frac{\exp(-ik_0R_{22})}{R_{22}}\Bigg|_{v_2'=v_0-L_2}^{v_2'=v_0+L_2} - \cos\Gamma\frac{dI_1(\eta_1')}{d\eta_1'}\frac{\exp(-ik_0R_{21})}{R_{21}}\Bigg|_{\eta'_1=-L_1}^{\eta'_1=L_1} +$$

$$+k_0^2\Psi_2\int_{-L_1}^{L_1} I_1(\eta_1')\frac{\exp(-ik_0R_{21})}{R_{21}}d\eta_1', \tag{4}$$

where ε_{eqm} is the equivalent permittivity,
$\varepsilon_{eqm} = \beta_m\cos^2\gamma_m + \sqrt{\varepsilon_1\beta_m}\sin^2\gamma_m$, where $\beta_m = \varepsilon_1\cos^2\gamma_m + \varepsilon_3\sin^2\gamma_m$,
$\alpha_m = -\dfrac{1}{2\ln\left(\dfrac{2L_m}{b_m}\dfrac{2\sqrt{\beta_m}}{\sqrt{\varepsilon_3}+\sqrt{\sigma_m}}\right)}$ is a small parameter, where $\sigma_m = \varepsilon_1\sin^2\gamma_m + \varepsilon_3\cos^2\gamma_m$,
$E_{0\eta 1}(\eta_1), E_{\eta 1}(\eta_1)$ are tangential components of source's field and total field on surface of first antenna,
$R_{11} = \sqrt{\beta_1(\eta_1 - \eta_1')^2 + r_1^2}$, where $r_m^2 = b_m^2\sqrt{\varepsilon_3\sigma_m}$ is equivalent radius of m-th antenna,
Γ is an angle between antennas,
$R_{12} = \sqrt{\beta_2(v_1 - v_2')^2 + d_{eq2}^2}$, where $d_{eq2}^2 = \varepsilon_3u_0^2 + \sigma_2w_0^2$,
$\Psi_1 = (\beta_1 - \varepsilon_{eq2})\cos\Gamma + (\varepsilon_3 - \varepsilon_1)\sin\gamma_1\cos\gamma_1\sin\Gamma$,

$E_{0v2}(v_2), E_{v2}(v_2)$ are tangential components of source's field and total field on surface of second antenna, $R_{22} = \sqrt{\beta_2 (v_2 - v_2')^2 + r_2^2}$, where $d_{eq1}^2 = \varepsilon_3 \chi_0^2 + \sigma_1 \xi_0^2$,

$\Psi_2 = (\beta_2 - \varepsilon_{eq1}) \cos\Gamma + (\varepsilon_3 - \varepsilon_1) \sin\gamma_2 \cos\gamma_2 \sin D$

$\sin D = \sqrt{\sin^2\varphi_2 \sin^2\gamma_1 + \cos^2\gamma_1 - \cos^2\Gamma}$,

χ_0, η_0, ξ_0 and u_0, v_0, w_0 are coordinates of the center of second antenna on systems $\chi\eta\xi$ and uvw accordingly.

Equations obtained in this way are solved by the method of averaging [3]. The advantage of the method of averaging is that it enables us to obtain uniform analytical expressions for the currents that are correct for antennas of any length, including resonant one. The obtained analytical expressions describe the currents in antennas of any length with positive and negative values of $\varepsilon_1, \varepsilon_3$. Rather simple expressions for currents are received at symmetric excitation of antennas. So for two symmetric active antennas excited by δ-generators, connected in the centres of antennas $E_{0\eta1}(\eta_1) = V_{01}\delta(\eta)$, $E_{0v2}(v_2) = V_{02}\delta(v - v_0)$ the expressions for currents look like

$$I_1(\eta_1) = V_{01} \frac{\sin k_1 (L_1 - |\eta_1|)}{Z_{11}} +$$

$$+ V_{02} \frac{Y_{12}(\eta_1) \sin 2k_1 L_1 - Y_{12}(L_1) \sin k_1 (L_1 + \eta_1)}{Z_{12}}, \quad (5)$$

$$I_2(v_2) = V_{02} \frac{\sin k_2 (L_2 - |v_2|)}{Z_{22}} +$$

$$+ V_{01} \frac{Y_{21}(v_2) \sin 2k_2 L_2 - Y_{21}(L_2) \sin k_2 (L_2 + v_2)}{Z_{21}}, \quad (6)$$

where

$k_n = k_n' + i k_n''$ is the complex wave numbers for m-th antenna, where m=1,2

$k_m = k_0 \left[\sqrt{\varepsilon_{eqm}} - \alpha_m \left(\sqrt{\varepsilon_1 \beta_m / \varepsilon_{eqm}} / k_0 b_m \right) (X_m - i R_m) \right]$,

functions Y_{12}, Y_{21} are integral functions describing mutual influence of antennas, Z_{11}, Z_{22} are own entrance resistance of antennas, Z_{12}, Z_{21} are mutual resistance .

All antenna characteristics were calculated for symmetrical antennas (7), (8) situated in the laboratory plasma with parameters: electron concentration 108 sm-3, constant magnetic field 3500 e, Langmuir frequency ω_N=5.6·108 Hz, Larmor frequency ω_B=6·1010 Hz. Under changing work frequency ω in range, where $\omega_B >> \omega >> \omega_N^2 / \omega_B$ plasma is described by di-

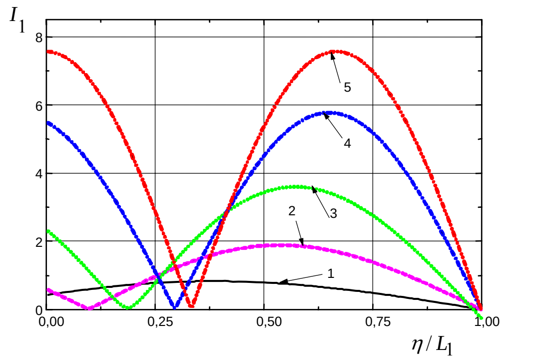

Fig. 2 Distribution of a current along one of antenna of system $L_1/\lambda = L_2/\lambda = 0{,}75$ depending on anisotropy of plasma $\varepsilon_3 = \{1$ - 0,1; 2 - 0,3; 3 - 0,5; 4 - 0,75; 5 - 1} at $d = \lambda$ and $\gamma = 90^0$.

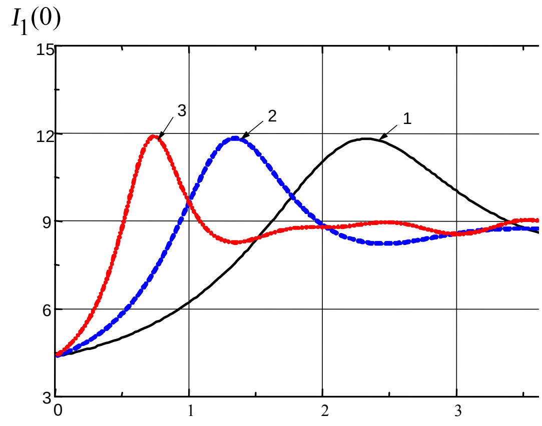

Fig. 3 Dependence of an input current in the first antenna (L_1/λ=0,75, L_2/λ =0,75) from distance between antennas at various ε_3 (1 -0,1; 2 -0,3; 3 -1) and γ=0^0.

agonal tensor ε. In this range the component ε_1 doesn't depend on frequency ($\varepsilon_1 \cong 1$), component ε_3 is function of frequency.
The functions of distribution of currents along antennas are determined by equivalent permittivity ε_{eqm} (Fig.2) and equivalent distance between antennas d_{eqm}, which depend on the values of components ε_1, ε_3 of permittivity tensor and on the orientation of antenna system in plasma. Parameter ε_{eqm} is various for each antenna, as it is determined not only by permittivity of plasma, but also orientation of antenna concerning a direction of magnetic field. Thus, the distribution of a current in antennas is established such, as though they work in various mediums.

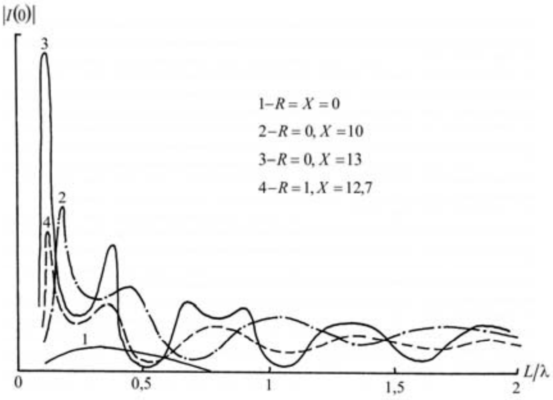

Fig.4. Dependence of input current of the antenna on its length and surface impedance $\varepsilon_1 = 1$, $\varepsilon_3 = -0{,}5$, $\gamma = 90^0$.

It is known, that for antennas located in isotropic medium, the amplitude of an input current of each antennas is reducing periodic function of distance between antennas. In anisotropic medium the character of this dependence is kept, but not for distance d, and for equivalent distance d_{eqm} (Fig. 3). For example, for antennas laying in one plane with an axis of anisotropy, d_{eqm} looks like $d_{eq}^2 = d^2\varepsilon_1\varepsilon_3(\varepsilon_1\cos^2\gamma + \varepsilon_3\sin^2\gamma)$. By each concrete antenna system with the given geometry in anisotropic medium can put in conformity some equivalent antenna system in anisotropic medium, choosing in appropriate way its geometrical sizes (this conclusion concerns only currents, instead of fields of radiation).
The influence of the surface impedance of antennas on the currents is characterised by variations of complex wave numbers $k_n = k_n' + ik_n''$, that include dielectric characteristics of medium, geometry of antennas and their orientation in magnetoactive plasma. The changes permittivity of plasma ε_1, ε_3, the orientations of the antenna (angle γ), imaginary part of a surface impedance X result in change of resonant length of the antenna, if ε_1, ε_3 positive.
In common case the conditions of effective excitation of the impedance antenna are:
a) a surface impedance should satisfy to conditions
$|\mathrm{Re}\,k| >> |\mathrm{Im}\,k|$, $|\mathrm{Im}\,k| \to 0$;
b) the antenna should be resonant
$L \cong (\lambda k_0/(4\,\mathrm{Re}\,k))(1+2n)$,
where n - integers;
c) the antenna should be not long, as the more than length of the antenna, the less amplitude of a current.
On Fig. 4 the input current of the impedance antenna is shown. The curve 1 corresponds to the perfect conducting antenna (it is not effective), curves 2,3,4 correspond to the impedance antenna (the input current is of a resonant character for short antenna). Let's notice, for curves 3 and 4 the conditions $\mathrm{Re}\,k_3 = \mathrm{Re}\,k_4$, $\mathrm{Im}\,k_4 > \mathrm{Im}\,k_3$ are satisfied.
Based on the distributions of currents according to the formulas (1), (2), it is possible to find the fields of radiation at any distance from antennas. The far field of radiation consists of two waves (ordinary and extraordinary waves) and looks like (in the spherical system of coordinates r, θ, φ)

$$\mathrm{E_r} = 0, \quad \mathrm{H_r} = 0,$$

$$E_\theta = \frac{k_0^2\sqrt{\varepsilon_1}\varepsilon_3}{i\omega N^3}\left[\sin\gamma_1\sin\phi\cos\theta - \cos\gamma_1\sin\theta\right]\Pi_{\mathrm{N1}}\frac{\exp(-ik_0 Nr)}{r} +$$
$$+\frac{k_0^2\sqrt{\varepsilon_1}\varepsilon_3}{i\omega N^3}\left[\sin\gamma_2\cos(\phi-\phi_2)\cos\theta - \cos\gamma_2\sin\theta\right]\Pi_{\mathrm{N2}}\frac{\exp(-ik_0 Nr)\exp(-ik_0 Nd\cos\alpha)}{r},$$

$$E_\phi = -\frac{k_0^2}{i\omega}\left[\sin\gamma_1\cos\phi\right]\Pi_{\mathrm{O1}}\frac{\exp\left(-ik_0\sqrt{\varepsilon_1}r\right)}{r} -$$
$$-\frac{k_0^2}{i\omega}\left[\sin\gamma_2\sin(\phi-\phi_2)\right]\Pi_{\mathrm{O2}}\frac{\exp\left(-ik_0\sqrt{\varepsilon_1}r\right)\exp\left(-ik_0\sqrt{\varepsilon_1}d\cos\alpha\right)}{r},$$

$$H_\phi = \frac{k_0^2\sqrt{\varepsilon_1}\varepsilon_3}{i\omega N^2}\left[\sin\gamma_1\sin\phi\cos\theta - \cos\gamma_1\sin\theta\right]\Pi_{\mathrm{N1}}\frac{\exp(-ik_0 Nr)}{r} +$$
$$+\frac{k_0^2\sqrt{\varepsilon_1}\varepsilon_3}{i\omega N^2}\left[\sin\gamma_2\cos(\phi-\phi_2)\cos\theta - \cos\gamma_2\sin\theta\right]\Pi_{\mathrm{N2}}\frac{\exp(-ik_0 Nr)\exp(-ik_0 Nd\cos\alpha)}{r},$$

$$H_\theta = \frac{k_0^2\sqrt{\varepsilon_1}}{i\omega}\left[\sin\gamma_1\cos\phi\right]\Pi_{\mathrm{O1}}\frac{\exp\left(-ik_0\sqrt{\varepsilon_1}r\right)}{r} +$$
$$+\frac{k_0^2\sqrt{\varepsilon_1}}{i\omega}\left[\sin\gamma_2\sin(\phi-\phi_2)\right]\Pi_{\mathrm{O2}}\frac{\exp\left(-ik_0\sqrt{\varepsilon_1}r\right)\exp\left(-ik_0\sqrt{\varepsilon_1}d\cos\alpha\right)}{r},$$

where γ_m, ϕ_m - angular coordinates of the *m*-th antenna, θ, ϕ - angular coordinates of the observer in spherical system of coordinates; $\mathrm{N}(\theta) = \sqrt{\varepsilon_3\sin^2\theta + \varepsilon_1\cos^2\theta}$; $\Pi_{\mathrm{N}m} = \int_{-L_m}^{L_m} I_m(\eta_m)\exp\left(ik_0 N(\theta)\eta_m\cos\Gamma_m\right)d\eta_m$; $\Pi_{\mathrm{O}m} = \int_{-L_m}^{L_m} I_m(\eta_m)\exp\left(ik_0\sqrt{\varepsilon_1}\eta_m\cos\Gamma_m\right)d\eta_m$; $I_m(\eta_m)$ is a current in the *m*-th antenna, $m = 1,2$. If in these formulas to put $\gamma_1 = \gamma_2 = \gamma$, we shall receive expressions for fields of radiation of parallel antennas, if to put $\gamma_2 = \gamma_1 + 90^0$, we shall find fields of perpendicular antennas. If in expressions for fields of radiation and currents to put $\varepsilon_1 = \varepsilon_3$, we receive the appropriate formulas for isotropic medium. By removing one of antennas on infinity, we shall receive the formulas for the lonely antenna.

It is known, that in frequency area which characterized by permittivity tensor, containing negative components, the feature of distribution of electromagnetic waves is determined by existence of cone-shaped zones of spread and non-spread of electromagnetic energy. Really, with $\varepsilon_1 < 0$, $\varepsilon_3 > 0$ the electromagnetic energy is spread only inside a resonant cone (according to the radiating diagram). With $\varepsilon_1 > 0$, $\varepsilon_3 < 0$ inside a resonant cone the energy is spread only in case of parallel orientation of the antenna concerning an axis of anisotropy. The impedance antenna, which orientation does not coincide with an axis of anisotropy can radiate energy in all directions, and not just in area limited to a resonant cone.

4. Conclusion

Anisotropy of medium essentially changes all characteristics of antennas. The functions of distribution of currents along antennas is determined by equivalent permittivity ε_{eq} and equivalent distance between antennas d_{eq}, which depend on the values of components $\varepsilon_1, \varepsilon_3$ of permittivity tensor and on the orientation of antenna system in medium (angle γ). Hence, these factors determine the shape of patterns. It is possible to make a conclusion about two ways of the design of antennas: electrical (changing the working frequency, that is $\varepsilon_1(\omega), \varepsilon_3(\omega)$) and mechanical (changing orientation in medium). It is known that if one of the components of permittivity tensor is negative, than with a certain orientation of antenna system relatively to the axis of anisotropy, the current in the perfectly conducting antenna not excited [4]. Excitation of significant currents in impedance antenna in this case is possible, provided that surface impedance is selected so that it compensates the reaction of surrounding plasma.

The received results can be useful at the decision of problems of high-frequency heating plasma, the analysis of electrodynamics compatibility of the devices working in plasma, and also to diagnostics of plasma.

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