Turbulent transport and flow effects on NTM evolution and trigger mechanisms

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Abstract. We study two problems related to the excitation and nonlinear evolution of neoclassical tearing modes in a tokamak, namely, (i) the effect of a background of microturbulence generated by short scale length instabilities such as the Ion Temperature Gradient (ITG) mode or the Electron Temperature Gradient (ETG) mode and (ii) the effect of a toroidal sheared flow on the stability of the $m = 1$ resistive internal kink mode that can provide the seed island trigger for an NTM. A background of ITG turbulence generates an anomalous viscosity whereas an ETG microturbulence leads to both anomalous current diffusivity and resistivity effects. The concommitant changes in the linear and nonlinear characteristics of a single helicity NTM are investigated analytically and expressed in terms of modifications of the Rutherford equations for the island evolution. The effects of the toroidal sheared flow on the resistive internal kink mode are investigated numerically using an initial value fully three dimensional toroidal code (NEAR) that solves a set of generalized reduced MHD equations. Shear flow is found to significantly modify both the linear growth rate and the nonlinear saturated island widths of the $m = 1$ mode.

1. Introduction

The excitation and nonlinear evolution of neoclassical tearing modes (NTM) is a subject of much current interest due to their potential deleterious impact on plasma confinement for long pulse experiments in superconducting tokamaks [1]. The size and lifetimes of the NTM saturated magnetic islands also set a limit on the plasma β and can therefore seriously compromise the efficiency of future reactor configurations. Theoretical and experimental efforts to gain a better understanding of the dynamics of NTMs and towards developing efficient means of controling their growth has therefore become an active and high priority area of research. While a great deal of progress has been achieved in delineating the basic features of this subcritical instability there are still a number of issues that are not well resolved. Among them are issues related to the creation of the seed island that triggers an NTM and the influence of shear flows and microturbulence on the excitation and growth processes. In this paper we report on our investigations on two problems related to these issues. Using an analytic approach based on the quasilinear theory, we first calculate the anomalous transport coefficients generated by fine scale microturbulence due to unstable ITG or ETG modes and then study the evolution of a single helicity NTM in the presence of such a background turbulence. The linear and nonlinear modifications in the characteristics of the NTM are obtained and their magnitudes estimated for realistic tokamak parameters. For the seed island problem the stability of an $m = 1$ resistive internal kink mode is numerically investigated using the 3d resistive MHD code NEAR in the presence of a sheared toroidal flow.

2. Turbulent transport effects on the NTM

Before we discuss the influence of turbulence induced anomalous transport phenomena on the evolution of an NTM we briefly recapitulate the essential physics governing the dynamics of this sub-critical instability. A neoclassical tearing mode instability arises due to the loss of the bootstrap current across a seed magnetic island located at a mode rational surface. The bootstrap current loss caused by a flattening of the pressure profile inside the island gives rise to a negative current perturbation that causes the island to grow further and thereby leads to an instability. The basic features of the instability can be obtained theoretically by including a bootstrap current source in the Ohm's law and thereafter carrying out the standard Rutherford analysis for a tearing mode [2]. This consists of evaluating the parallel current contribution $J_{\|}$ and then using it in the matching conditions,

$$\int_{-\pi}^{\pi} d\xi cos\xi \int_{-\infty}^{\infty} dx J_{\|} = \frac{c}{4\pi}\Delta'_c \pi\tilde{\psi} \tag{1}$$

$$\int_{-\pi}^{\pi} d\xi sin\xi \int_{-\infty}^{\infty} dx J_{\|} = \frac{c}{4\pi}\Delta'_s \pi\tilde{\psi} \tag{2}$$

to obtain the island evolution equation. Here ψ is the total poloidal flux function given by,

$$\psi = -x^2 \frac{B_0}{2L_s} + \tilde{\psi}(t) cos\xi \tag{3}$$

where B_0 is the average equilibrium toroidal magnetic field, $x = r - r_s$ is the distance from the rational surface, $L_s = qR/s$ is the shear length, $s = r_s q'/q$ and q is the safety factor. Further $\xi = m\hat{\theta} - \int \omega(t')dt'$, $\hat{\theta} = \theta - \zeta/q_s$ is the helical coordinate with θ denoting the poloidal angle, ζ the toroidal angle and $q_s = m/n$ is the value of q at the mode rational surface (m and n are the poloidal and toroidal mode numbers of the helical perturbed flux function $\tilde{\psi}$). For $m \geq 2$, when the constant $\tilde{\psi}$ approximation holds, the magnetic island halfwidth is given by,

$$W = \left(\frac{4L_s\tilde{\psi}}{B_0} \right)^{1/2} \tag{4}$$

The modified Ohm's law is given by,

$$\eta J_{\|} = -\nabla_{\|}\tilde{\phi} + \frac{1}{c}\frac{\partial\tilde{\psi}}{\partial t} cos(\xi) - \eta J_b \tag{5}$$

where

$$J_b = \frac{\mu_e}{\nu_{ei}} \frac{c}{B_\theta} \frac{dp}{dx}$$

the perturbed bootstrap current is the driving source of the instability. Here μ_e is the viscosity coefficient, ν_{ei} is the electron-ion collision frequency, B_θ is the poloidal magnetic field and p is the plasma pressure. In the simplest case where inertial effects are neglected the quasineutrality condition $\nabla \cdot \vec{J} = 0$ can be approximated by $(\vec{B} \cdot \nabla) J_{\|} \approx 0$ which implies that $J_{\|}$ is a flux function, i.e. $J_{\|} = J_{\|}(\psi)$. Following the standard procedure [2, 3] one can then obtain an island evolution equation in the form,

$$G_1 \frac{\partial W}{\partial t} = D_R \left[\frac{\Delta'_c}{4} + G_2 \frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W} \frac{W^2}{W^2 + W_\chi^2} \right] \tag{6}$$

where, $D_R = \eta c^2/4\pi$, $\beta_\theta = 8\pi p_e/B_\theta^2$, $L_p = -(dlnp/dr)^{-1}$ and $L_q = (dlnq/dr)^{-1}$. The coefficients G_1,G_2 are constants and the W_χ term appears from consideration of finite perpendicular thermal conductivity inside the island. It defines a threshold island width for the driving bootstrap term to overcome the stabilizing term ($\Delta'_c < 0$) and consitutes the "seed island" necessary for the instability to occur. The above minimal form of the modified Rutherford equation for the NTM instability can be improved upon by the inclusion of additional contributions arising from curvature effects, polarization current terms etc which can influence both the excitation threshold as well as the saturation levels of the instability. In the following two subsections we will study the influence of a background microturbulence generated respectively by the ITG and ETG instabilities on the evolution characteristics of a single helicity NTM and express them in terms of modifications of the modified Rutherford equation.

2.1. NTM in an ITG turbulence

The wide scale separation in the temporal and spatial characteristics of an NTM mode and those of the ITG and ETG modes permits a multiple scale based analytic study of the mutual inteactions between them. We exploit the fact that there is a sufficiently large non-axisymmetry in the wave spectrum, which separates the long scale perturbation $(\gamma_q, \vec{\mathbf{q}})$ of the slowly growing NTM from the short scale $\left(\omega, \vec{k}\right)$ perturbations of the ion temperature gradient (ITG) or the electron temperature gradient (ETG) driven turbulence. The evolution of the NTM can be obtained from a set of model equations consisting of the vorticity equation, the quasineutrality condition $\left(\vec{\nabla} \cdot \vec{\mathbf{J}} = 0\right)$ and the parallel electron momentum equation (Ohm's law). To account for the background ITG turbulence one includes the slow time and long wavelength response of the nonlinear terms in these equations. The model equations can then be written in the form,

$$\frac{d^{(i)}}{dt}\nabla_\perp^2\tilde{\phi}_q + \nabla_\parallel\nabla_\perp^2\tilde{A}_{\parallel q} = -\left\langle\left[\phi_k^{ITG}, \nabla_\perp\phi_k^{ITG}\right]\right\rangle - \left\langle\vec{\nabla}\cdot\left[\phi_k^{ITG}, \vec{\nabla}p_{ik}^{ITG}\right]\right\rangle \tag{7}$$

$$\frac{\beta}{2}\frac{\partial}{\partial t}\tilde{A}_{\parallel q} + \nabla_\parallel\tilde{\phi}_q - \hat{\eta}\nabla_\perp^2\tilde{A}_{\parallel q} = \hat{\eta}J_b \tag{8}$$

The right hand side of Eq.(7) consists of the Reynold stress contributions representing the quadratic nonlinear interactions of the background ITG modes [4], with $< .. >$ representing averaging over the fast time scales and [,] denoting a Poisson bracket. The other parameters are normalized as,

$$\tilde{\phi}_q = \frac{L_n}{\rho_s}\frac{e\delta\phi_q}{T_e},\ \ \tilde{A}_{\parallel q} = \frac{e\tilde{\psi}}{T_e}\frac{2c_sL_n}{c\beta\rho_s},\ \ t = t'c_s/L_n,\ \ \nabla_\perp \to \rho_s\nabla_\perp,\ \ \nabla_\parallel \to L_n\nabla_\parallel,$$

$$\hat{\eta} = \frac{0.51\nu_eL_n}{C_e}\sqrt{\frac{m_e}{m_i}},\ \ \frac{d^{(i)}}{dt} = \frac{\partial}{\partial t} + \alpha_i\frac{\partial}{\partial y},\ \ \alpha_i = (1+\eta_i)\frac{T_i}{T_e}.$$

We adopt a quasilinear approach for calculating the nonlinear terms on the right hand side of Eq.(7). In the limit $q_x > q_y$, Eq.(7) can be rewritten as,

$$\frac{d^{(i)}}{dt}\nabla_\perp^2\tilde{\phi}_q + \nabla_\parallel\nabla_\perp^2\tilde{A}_{\parallel q} = \nabla_x^2\int d^3k\left[1+\tau_i(1+\Lambda_0)\right]k_xk_y|\phi_k^{ITG}|^2 \tag{9}$$

where,

$$\Delta_0 = \frac{\Delta_1\Delta_2 - 2\gamma_k^2/3}{\Delta_2^2+\gamma_k^2},\ \ \Delta_1 \simeq (\eta_i - 2/3)k_y,\ \ \Delta_2 \simeq \frac{k_y}{2}(1-\epsilon_n),$$

$$\gamma_k^{ITG} \simeq \frac{k_y}{2}\sqrt{\tau_i \epsilon_n}\left(\eta_i - \eta_{th}\right)^{1/2}, \quad \eta_{th} \simeq \frac{2}{3} - \frac{1}{2\tau_i} + \epsilon_n\left(\frac{1}{4\tau_i} + \frac{10\tau_i}{9}\right) + \frac{1}{4\tau_i\epsilon_n},$$

$$\eta_i = L_n/L_T, \quad \omega_r \simeq a - bk_\perp^2,$$

$$a = \frac{k_y}{2}\left[1 - \left(1 + \frac{10\tau_i}{3}\right)\epsilon_n\right], \quad b = \frac{k_y}{2}\left[1 + \tau_i(1+\eta_i) - \epsilon_n\left(1 + \frac{5}{3}\tau_i\right)\right]$$

In the absence of any source, sinks and slow variation terms the total energy is conserved. Thus we can write an adiabatic invariant, $N_k = E_k/\omega_k$ where E_k is the energy of the k_{th}-mode and ω_k is the frequency. The response of the tearing mode perturbations on the ITG turbulence can be calculated from a wave kinetic equation [5],

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega}{\partial \vec{k}}\frac{\partial N_k}{\partial X} - \frac{\partial}{\partial \vec{X}}(\omega + k\cdot\tilde{V}_E)\frac{\partial N_k}{\partial \vec{k}} = \gamma N_k - \Delta\omega N_k^2 \tag{10}$$

Here $k\cdot\tilde{V}_E$ is the effective doppler shift from the slowly varying perturbation. The nonlinear term in Eq.(7) is a function of N_k via $\delta|\phi|^2 = (\delta|\phi_k|^2/\delta N_k)\,\delta N_k = \Delta_*^{ITG}\delta N_k$. From (10) the response of the slowly varying field is ,

$$\delta N_k = \frac{k_y}{(\gamma_k^{ITG} - i\Omega_q + iq_x V_{gx})}\frac{\partial^2\tilde{\phi}_q}{\partial X^2}\frac{\partial N_0}{\partial k_X} \tag{11}$$

Substituting Eq.(11) into Eq. (9) we get,

$$\frac{d^{(i)}}{dt}\nabla_\perp^2\tilde{\phi}_q + \nabla_\parallel\nabla_\perp^2\tilde{A}_{\parallel q} = \mu_i^{an}\frac{\partial^4\tilde{\phi}_q}{\partial x^4} \tag{12}$$

where,

$$\mu_i^{an} = \int d^3k\,[1 + \tau_i + \tau_i\Gamma_0]\,\frac{k_y^2 k_x \gamma_k^{ITG}}{(\gamma_k^2 + q_x^2 V_{gx}^2)\Delta_*^{ITG}}\left(-\frac{\partial N_0}{\partial k_x}\right)$$

and we have retained only the leading contribution on the right hand side. Thus the presence of a background ITG turbulence gives rise to an anomalous viscosity coefficient μ_i^{an}. To assess the impact of this term on the nonlinear evolution of the NTM we need to solve (12) for $J_\parallel(=\nabla_\perp^2\tilde{A}_{\parallel q})$ and use it in the matching conditions (1) and (2). Carrying out this procedure we find that the term does not contribute to the island growth equation (i.e. to matching condition (1)) due to symmetry considerations but makes a finite contribution to the matching condition (2) which gives an island rotation equation of the form,

$$\frac{\partial}{\partial t}\left[W(\omega - \omega_E)\right] = -G_3\frac{\mu_i^{an}}{W}(\omega - \omega_E) - G_4\left(\frac{nsV_A}{R^2 q}\right)^2 W^4\Delta_s' \tag{13}$$

where ω is here the island rotation frequency and ω_E is the poloidal rotation frequency due to the perturbed radial electric field. We estimate the magnitude of anomalous viscosity by taking an average saturated amplitude of the background ITG turbulence from mixing length arguments to be $\frac{e\phi^{ITG}}{T_e}\frac{L_n}{\rho_s} \simeq 1$,. This yields $\mu_i^{an} \simeq (1 + \tau_i + \tau_i\Gamma_0)\frac{k_y\rho_s}{\sqrt{\tau_i\epsilon_n(\eta_i - \eta_t h)}}$. This additional viscous drag due to the background turbulence can enhance the island braking process and contribute to a mode locking process.

2.2. NTM in an ETG Turbulence

We next investigate the nonlinear evolution of a NTM in a background of ETG mode turbulence. The equations for the slow, long scale NTM are once again the vorticity and parallel electron momentum equations. When averaged over fast time and space scales, these equations become:

$$\frac{d^{(i)}}{dt}\nabla_\perp^2\tilde{\phi}_q + \nabla_\parallel\nabla_\perp^2\tilde{A}_{\parallel q} = -\left\langle\left[\phi_k^{ETG}, \nabla_\perp^2\phi_k^{ETG}\right]\right\rangle + \frac{\beta}{2}\left\langle\left[A_{\parallel k}^{ETG}, \vec{\nabla}_\perp^2 A_{\parallel k}^{ETG}\right]\right\rangle \tag{14}$$

$$\frac{\beta}{2}\frac{d^{(e)}}{dt}\tilde{A}_{\parallel q} - \frac{m_e}{m_i}\frac{\partial}{\partial t}\nabla_\perp^2\tilde{A}_{\parallel q} + \nabla_\parallel\left(\tilde{\phi}_q - \tilde{p}_{iq}\right) - \hat{\eta}\nabla_\perp^2\tilde{A}_{\parallel q} - \hat{\eta}J_b =$$
$$\frac{\rho_e}{\rho_s}\frac{\beta}{2}\left\langle\left[A_{\parallel k}^{ETG}, (\phi_k^{ETG} - p_k^{ETG})\right]\right\rangle + \frac{\rho_e}{\rho_s}\left\langle\left[\phi_k^{ETG}, \nabla_\perp^2 A_{\parallel k}^{ETG}\right]\right\rangle \tag{15}$$

where $\frac{d^{(e)}}{dt} = \left(\frac{\partial}{\partial t} + \alpha_e\frac{\partial}{\partial y}\right)$. In the above equations space, time and perturbed fields associated with the NTM mode are normalized as in the previous section whereas the nonlinear terms arising from the coupling between the NTM and the ETG modes [6] are normalized as,

$$k_\perp^{ETG} \to k_\perp^{ETG}\rho_e, \quad \tilde{\phi}_k^{ETG} \to \frac{e\delta\phi}{T_e}\frac{L_n}{\rho_e}, \quad t \to t' L_n/c_s,$$
$$\omega \to \omega L_n/c_e, \quad A_{\parallel k}^{ETG} \to \frac{e\delta A_\parallel}{T_e}\frac{c_e}{c}\frac{L_n}{\rho_e}\frac{2}{\beta}, \quad \nabla_\parallel \to L_n\nabla_\parallel,$$

where ρ_e and c_e are the Larmor radius and thermal velocity of the electrons respectively and $\alpha_e = 1+\eta_e$. Note that in contrast to the ITG case, nonlinear interaction terms appear in the Ohm's law as well due to finite electron inertia effects and can thereby impact the NTM dynamics in a significant way. We can estimate the nonlinear contributions once again through the quasilinear approach as described in the previous section. Before doing that we list below the linear relation between the various fields of the ETG mode [6],

$$\tilde{A}_{\parallel k}^{ETG} = k_\parallel\frac{\left[\omega - \frac{10}{3}\epsilon_n k_y - (1+\eta_e) - \frac{5}{3}(1-\tau)k_y\right]}{\left[\frac{\beta}{2}\left\{\omega - (1+\eta_e)k_y\right\} + k_y^2\omega\right]\left[\omega - \frac{10}{3}\epsilon_n k_y\right]}\tilde{\phi}_k^{ETG} \equiv R_A\tilde{\phi}_k^{ETG}$$
$$\tilde{p}_k^{ETG} \approx \frac{\left[1+\eta_e - \frac{5}{3}\epsilon_n(1-\tau)\right]k_y}{\omega - \frac{10}{3}\epsilon_n k_y} \equiv R_p\phi_k^{ETG}$$

The real frequency and the linear growth rate of the ETG mode are,

$$\omega_{r0} \approx \frac{k_y}{2(\tau_e + k_\perp^2)}\left[1 - \epsilon_n(1 + \frac{10}{3}\tau_e)\right]$$
$$\gamma_0^{ETG} \approx \frac{k_y}{(1+k_\perp^2)}\sqrt{\epsilon_n\tau_e(\eta_e - \eta_{th})};$$
$$\eta_{th} = \frac{2}{3} - \frac{1}{2\tau_e} + \frac{1}{4\epsilon_n\tau_e} + \epsilon_n\left(\frac{1}{4\tau_e} + \frac{10}{9\tau_e}\right)$$

Including the parallel electron motion perturbatively results in a shift of the real frequency and a stabilizing effect on the growth rate. For $\beta < 1$, the magnitude of the frequency shift and the stabilizing contribution due to parallel motion are

$$\omega_1 = \frac{k_y k_\parallel^2}{\omega_{r0}^2 + r_o^2}\left(\eta_e - \frac{2}{3} + \frac{10}{3}\epsilon_n\right) \tag{16}$$

$$\gamma_1 = -\frac{k_\parallel^2}{2\gamma_0}\left[\frac{8}{3} - \frac{k_y\omega_{r0}}{\omega_{r0}^2 + r_0^2}\left(\eta_e - \frac{2}{3} + \frac{10}{3}\epsilon_n\right)\right] \tag{17}$$

Using these linear responses, the nonlinear terms in Eqns. (14) – (15) can be expessed in terms of $|\tilde{\phi}_k^{ETG}|^2$. In the limit $q_x > q_y$ and $\beta/2 > \frac{m_e}{m_i}\rho_s^2 k_\perp^2$. Eqns. (14)-(15) can be rewritten as,

$$\left(\frac{\partial}{\partial t} + \alpha_i\frac{\partial}{\partial y}\right)\nabla_x^2\tilde{\phi}_q + \nabla_\parallel\nabla_x^2\tilde{A}_{\parallel q} = \frac{\rho_e^2}{\rho_s^2}\nabla_x^2\int d^3k k_x k_y\left(1 - \frac{\beta}{2}|R_A|^2\right)|\tilde{\phi}_k^{ETG}|^2 \tag{18}$$

$$\frac{\beta}{2}\frac{d^{(e)}}{dt}\tilde{A}_{\parallel q} - \hat{\eta}\nabla_\perp^2\tilde{A}_{\parallel q} + \nabla_\parallel(\tilde{\phi}_q - \tilde{p}_q) - \hat{\eta}J_b = -\frac{\rho_e^2}{\rho_s^2}\nabla_x^2\int d^3k k_x k_y R_A^R|\tilde{\phi}_k^{ETG}|^2$$

$$+\frac{\rho_e}{\rho_s}\nabla_x\int d^3k k_\perp^2 k_y R_A^I|\tilde{\phi}_k^{ETG}|^2 + \frac{\rho_e}{\rho_s}\frac{\beta}{2}\nabla_x\int d^3k k_y\left[1 - R_p^R\left\{1 - \frac{R_p^I R_A^R}{R_p^R R_A^I}\right\}\right]R_A^I|\tilde{\phi}_k^{ETG}|^2 \tag{19}$$

The action density $N_k \propto |\phi_k^{ETG}|^2$ is the adiabatic invariant that couples to the slow NTM via the wave kinetic equation (WKE). The real frequency and growth rate expressions in the WKE are modified by the presence of the slow mode. The effective shift of real and imaginary frequencies from the slowly varying field can be written as

$$\delta\omega_1 = \vec{k}_\perp\cdot\left[z\times\vec{\nabla}\tilde{\phi}_q - \frac{\beta}{2}v_{g\parallel}\hat{z}\times\vec{\nabla}\tilde{A}_{\parallel q}\right] \tag{20}$$

$$\delta\gamma_1 = -\frac{\beta}{2}\left(\frac{\partial\gamma_k^{ETG}}{\partial k_z}\right)\vec{k}_\perp\cdot\hat{z}\times\vec{\nabla}\tilde{A}_{\parallel q} \tag{21}$$

Here $V_{g\parallel} = \partial\omega_1/\partial k_\parallel$ and $\partial\gamma_{k1}^{ETG}/\partial k_\parallel$ can be calculated from Eq.(16) and (17) and $\tilde{A}_{\parallel q}$, the slow perturbation results from the modulation of frequency and growth rate of ETG via $k_\parallel$ modulation, where $\delta k_\parallel \to \vec{z}\times\vec{\nabla}\tilde{A}_{\parallel q}/B$. The linearized wave kinetic equation can now be written as

$$\frac{\partial}{\partial t}\delta N_q + V_g\cdot\nabla\delta N_q - k_y\nabla_x^2\left(\tilde{\phi}_q - \frac{\beta}{2}V_{g\parallel}\tilde{A}_{\parallel q}\right)\frac{\partial N_0}{\partial k_x} \simeq -\gamma_k\delta N_k - \frac{\beta}{2}\frac{\partial\gamma_{k1}}{\partial k_\parallel}k_y\nabla_x\tilde{A}_{\parallel q}N_0 \tag{22}$$

and which can be solved to give,

$$\delta N_q = k_y\nabla_x^2\left(\tilde{\phi}_q - \frac{\beta}{2}v_{g\parallel}\tilde{A}_{\parallel q}\right)\frac{\partial N_0}{\partial k_x}R(qv_{gx}) - k_y\frac{\beta}{2}\frac{\partial\gamma_{k1}}{\partial k_\parallel}\nabla_x\tilde{A}_{\parallel q}N_0 R(q_x v_{gx}) \tag{23}$$

where $R(qv_{gx}) = (\gamma_k + iq_x v_{gx} + \gamma_q)^{-1}$. Substituting (23) into Eqs. (18) and (19), the governing equations for the NTM then take the form:

$$\frac{d^{(i)}}{dt}\nabla_x^2\tilde{\phi}_q + \nabla_\parallel\nabla_x^2\tilde{A}_{\parallel q} = -\mu_{e\perp}\nabla_x^4\tilde{\phi}_q \tag{24}$$

$$\frac{\beta}{2}\frac{d^{(e)}}{dt}\tilde{A}_{\parallel q} - \hat{\eta}\nabla_\perp^2\tilde{A}_{\parallel q} + \nabla_\parallel\tilde{\phi}_q + \hat{\eta}J_b = -\mu_\parallel^{an}\nabla_x^4\tilde{A}_{\parallel q} + \mu_B^{an}\nabla_x^2\tilde{A}_{\parallel q} \tag{25}$$

where

$$\mu_{e\perp} = \left[\frac{\rho_e^2}{\rho_s^2}\int d^3k k_y^2 k_x\left(1 - \frac{\beta}{2}|R_A|^2\right)Re(q_x v_{gx})\left(-\frac{\partial N_0}{\partial k_x}\right)\right]$$

$$\mu_\parallel^{an} = \left[\frac{\beta}{2}\frac{\rho_e^2}{\rho_s^2}\int d^3k k_y^2 k_x R_A^I v_{g\parallel}\left(-\frac{\partial N_0}{\partial k_x}\right)R(qv_{gx})\right]$$

$$\mu_B^{an} = \left[\frac{\beta}{2} \frac{\rho_e}{\rho_s} \int d^3 k k_y^2 \left(-\frac{\partial \gamma_{k1}}{\partial k_\parallel} \right) \left\{ k_\perp^2 + \left(1 - R_p^R \left\{ 1 - \frac{R_p^I R_A^R}{R_p^R R_A^I} \right\} \right) \right\} \times R_A^I R(q_x v_{gx}) N_0 \right]$$

Here $\mu_\parallel^{an}, \mu_B^{an} > 0$ represent anomalous current diffusivity and resistivity arising from the ETG turbulence effects. A standard Rutherford analysis of the coupled equations (24) and (25) leads to a modified island evolution equation of the form,

$$G_5 \frac{W^2}{W^2 + W_{an}^2} \frac{\partial W}{\partial t} = D_R^{an} \left[\frac{\Delta_c'}{4} + G_6 \frac{\sqrt{\epsilon} \beta_\theta \frac{L_q}{L_p}}{W} \frac{W^2}{W^2 + W_{an}^2} \right] \tag{26}$$

where G_5, G_6 are constants, $D_R^{an} = D_R + \mu_B^{an} c^2 / 4\pi$ and W_{an} is proportional to $\mu_\parallel^{an} / \hat{\eta}$. Thus we find that the anomalous current diffusivity arising from ETG turbulence gives rise to an effect that is similar to perpendicular thermal conductivity and defines a new threshold for the onset of the NTM instability [7, 8]. The effect is most pronounced at small island widths. The μ_B^{an} contribution acts like an anomalous resistivity effect and adds to the classical resistivity term. It can therefore influence both the linear as well as the nonlinear growth rate of the island.

3. Sheared flow effects on the resistive internal kink mode

In this section we briefly report on our preliminary investigation of the effect of a sheared toroidal flow on the linear and nonlinear characteristics of the resistive internal kink mode. The overall objective is to assess the impact of such a flow on the excitation and size of the "seed" island that acts as a trigger for the NTM. Many experimental observations have seen a strong correlation between a large sawteeth crash and the emergence of a seed island with the subsequent development of an NTM instability. Since the internal resistive kink instability is intimately involved in the sawtooth phenomenon we focus our attention on the evolution of such an instability in the presence of flow. Our study is primarily numerical and is based on the solution of a set of generalized reduced MHD equations that have been used in the past for NTM studies [9]. A fully toroidal three dimensional code called NEAR is employed for this purpose. After generating an appropriate equilibrium with flow in another independent code called TOQ [10], the time evolution of an $m = 1$ internal kink mode is studied in NEAR for this equilibrium. The initial q profile is chosen such that $q(0) < 1$ so that the internal kink mode is unstable. The effect of both differential flow and flow shear are studied by choosing different toroidal flow profiles. Our preliminary results are displayed in Fig. 1. For both differential as well as shear flow profiles [9] we find that the linear growth of the kink mode is reduced as a function of the flow magnitude. However as the mode evolves nonlinearly and eventually saturates the presence of flow tends to increase the saturated island width. More detailed parametric studies including the effect of varying the resistivity and viscosity are currently in progress for a better understanding of the nonlinear dynamics.

4. Summary and Discussion

To summarize, we have carried out a model calculation to assess the effect of a background microturbulence of ITG or ETG modes on the linear and nonlinear characteristics of a single helicity neoclassical tearing mode. A quasilinear self-consistent analysis shows the generation of anomalous viscosity terms in the case of ITG turbulence which can cause an enhanced slowing down of the rotation of the mode. A background of ETG turbulence can

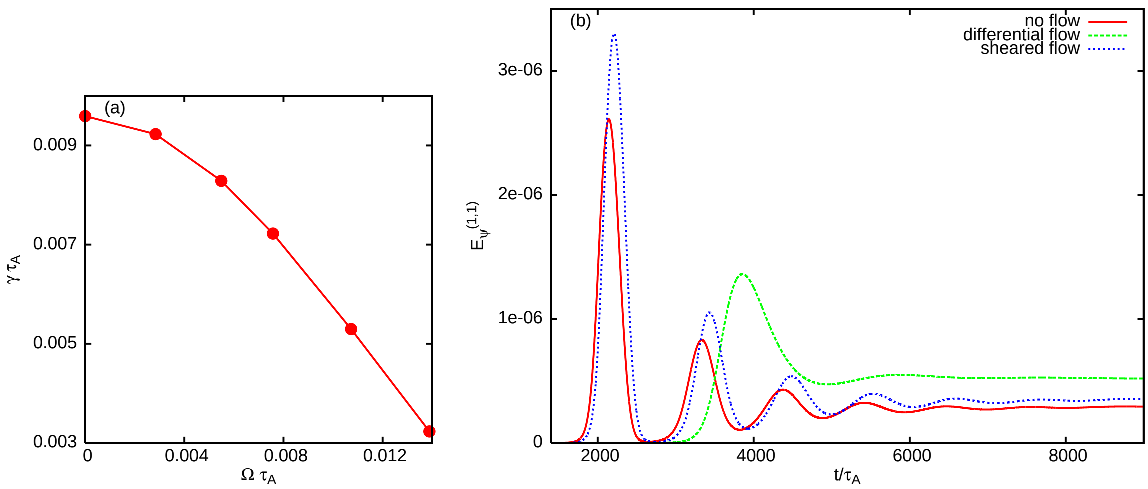

Figure 1. (a) The reduction in the linear growth rate of the resistive internal kink mode as a function of the differential flow amount. (b) A comparison of the nonlinear characteristics of the mode in the absence and presence of differential and sheared flows.

provide anomalous resistive effects which can influence the linear and nonlinear growth of the NTM. In addition it also creates an anomalous current diffusivity which introduces a new seed island threshold for the excitation of the NTM in a manner analogous to finite thermal conduction effects. We have also carried out a preliminary numerical investigation of the effect of a toroidal equilibrium flow on the development of an internal resistive kink mode and found significant flow induced modifications in the linear and nonlinear characteristics of the mode. This can have important implications for the excitation of a "seed" island normally associated with a sawtooth crash.

References

[1] T. Hender *et al*, *Progress in the ITER Physics Basis, Chapter 3: MHD Stability, operational limits and disruptions*, Nuclear Fusion **47**, S128 (2007).
[2] C.C. Hegna, Phys. Plasmas **5**, 1767 (1998).
[3] A.I. Smolyakov, A. Hirose, E. Lazzaro, G.B. Re and J.D. Callen, Phys. Plasmas **2**, 1581 (1995).
[4] C.J. McDevitt and P. H. Diamond, Phys. Plasmas 13, 0232302 (2006).
[5] P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm, Plasma Phys. Controlled Fusion 47, R35 (2005).
[6] R. Singh, P. K. Kaw and J. Weiland, Nuclear Fusion 41, 1219 (2001).
[7] P. K. Kaw, E. J. Valeo, and P. H. Rutherford: Phys. Rev. Lett. **26** (1971) 753.
[8] A. Smolyakov, P. Diamond, M. Yagi, K. Itoh, and S. Itoh, J. Phy. Soc. Japan, **76**, 113501 (2007).
[9] D. Chandra, A. Sen, P. Kaw, M.P. Bora and S. Kruger, Nucl. Fusion **45**, 524 (2005).
[10] http://fusion.gat.com/toq