Weibel Instability in a Bi-Maxwellian Laser Fusion Plasma

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Abstract. We are interested in this paper to analyse the Weibel instability driven by the plasma temperature anisotropy in the corona of a high intense laser created plasma. The unperturbed electronic distribution function, f, of the anisotropic corona is supposed to be a bi-maxwellian. That $T_{//} = T_{\perp} + W_O$, where $W_O = \frac{1}{4} m_e v_O^2$ is the averaged electron quiver energy in the laser electric field. The first and the second anisotropies of f projected on the Legendre polynomials are calculated as function of the scaling parameter, $\frac{W_O}{T_{\perp}}$. The Weibel instability parameters are explicitly calculated as function of the scaling parameter. For typical parameters of the laser pulse and the fusion plasma, it has been shown highly unstable Weibel modes: $\gamma \approx 10^{11} s^{-1}$ excited in the corona.

1. Introduction

In the inertial confinement fusion (ICF) targets, produced by an intense laser pulse, the incident laser wave produces an anisotropy in the formed plasma temperature. This is due to the fact that the formed plasma is preferentially heated in the direction of the laser wave electric field. This temperature anisotropy can be interpreted, in the frame of the kinetic theory, as an anisotropy in the electrons velocities distribution [1,2,3]. It has been shown that this anisotropic distribution provokes unstable Weibel electromagnetic modes [4,5,6].

If this instability is excited, the target may have a possibility to give rise to energy loss. The implosion characteristics of the target are influenced. Giga gauss magnetic fields due to this instability can be generated in the plasma corona.

This paper deals with the theoretical study of the Weibel instability excited in the laser fusion plasma corona. In our model the unstable Weibel modes are exited in the corona by the direct effect of the laser electric field on the coronal plasma. That the corona which is characterized by a plasma frequency, ω_P, less than the laser wave frequency, ω_L : $\omega_P < \omega_L$, is the direct interaction region between the incident laser pulse and the formed plasma.

The present work is organized as follows: in section 2., we present the electronic distribution function which is supposed to be a local bi-Maxwellian. In section 3., we present a theoretical analysis of the Weibel modes. The section 4. is devoted to the scaling laws for the instability parameters. Finally, a conclusion for the obtained results is given.

2. Distribution function

In our model, the electronic distribution function, f, is supposed to be a local bi-Maxwellian.

$$f = \left(\frac{m_e}{2\pi}\right)^{3/2} \frac{n_e}{T_\perp T_\parallel^{1/2}} \exp\left(-\frac{1}{2}\frac{m_e v_\perp^2}{T_\perp}\right) \exp\left(-\frac{1}{2}\frac{m_e v_\parallel^2}{T_\parallel}\right), \tag{1}$$

where $e, m_e, n_e, T_\parallel, T_\perp$ are respectively the elementary electric charge, the electron mass, the electrons density, the parallel temperature to the anisotropy direction and the temperature in the perpendicular plane.

In the case of the linear polarized laser pulse, with an electric laser wave oriented in the parallel direction: $T_\parallel = T_\perp + W_O$, where W_O is the average, on the laser cycle, of the oscillating energy communicated to the electron by the laser wave. But in the case of the circularly polarized laser wave, with an electric laser wave oscillating in the perpendicular plane: $T_\parallel = T_\perp - W_O$.

The average quiver energy, W_O , is calculated using the perturbed fluid electron motion equation by considering the collisions [7,8,9,10], so:

$$W_O = \frac{e^2}{8\varepsilon_0 c m_e} \frac{I}{\omega_L{}^2} \left(1 - \frac{1}{2}\left(\frac{\nu_c}{\omega_L}\right)^2\right), \tag{2}$$

where $\varepsilon_{0,}$ c, I and $\nu_c \sim n_e/(T)^{3/2}$ are respectively the vacuum electric permittivity, the speed of light in the vacuum, the laser pulse intensity and the collisions frequency, where $T = \int_0^\infty d^3\vec{v} f \frac{m_e}{2} v^2 \,/\, \int_0^\infty d^3\vec{v} f\, v^2 dv$ is the electrons temperature.

We develop the bi-Maxwellian electronic distribution function (eq. 1) on the Legendre polynomials, $P_l(\mu = \frac{v_\parallel}{v})$ [11]: $f = \sum_0^\infty P_l(\mu) f_l(v)$. Hence, the isotropic distribution function, f_0, the first, f_1 and the second, f_2 , are calculated as:

$$f_0 = \left(\frac{m_e}{2\pi T_\perp}\right)^{3/2} n_e \exp(-y) \left\{\left(1 + \widetilde{W_o}\right)^{-\frac{1}{2}} + \frac{y}{3}\widetilde{W_o}\left(1 + \widetilde{W_o}\right)^{-\frac{3}{2}}\right\}, \tag{3}$$

$$f_1 = 0, \tag{4}$$

$$f_2 = \left(\frac{m_e}{2\pi T_\perp}\right)^{3/2} n_e \exp(-y) \left\{\frac{2y}{3}\widetilde{W_o}(1 + \widetilde{W_o})^{-1/2}\right\}, \tag{5}$$

where $y = \frac{m_e v^2}{2T_\perp}$ and $\widetilde{W_o} = \frac{W_o}{T_\perp}$.

We have presented, on the FIG.1., the $f_0(y)$, for several values of the scaling parameter $\widetilde{\mathrm{W}}_0$.

3. Weibel instability analysis

In the reference [6], a dispersion relation, in the semicollisional regime, is established. This dispersion relation is derived in the Lorentz gas and in the local approximations. It is valid in the whole collisionality regime. Practical expression of the growth rate of the most unstable Weibel mode and its group velocity are computed from this relation, so:

$$\gamma_{max} = \frac{2^{15/4}}{3^{3/2}\sqrt{\pi}} \frac{v_t^{5/2}}{\sqrt{n_e}} \frac{\omega_p}{c} \frac{\left[\int_0^\infty \sqrt{y} f_2 dy\right]^{3/2}}{\int_0^\infty f_0 dy}, \tag{6}$$

$$v_g = \frac{v_t}{\sqrt{2}} \frac{\int_0^\infty \sqrt{y} f_1 dy}{\int_0^\infty f_0 dy}, \tag{7}$$

where $v_t = \sqrt{T/m_e}$ is the electrons thermal velocity.
By considering the explicit expressions of f_1 *and* f_2 (eqs. 3-5), the above equations can be written as function of the scaling parameter, $\widetilde{W_o}$, so:

$$\frac{\gamma_{max}}{10^{11}s^{-1}} = 4.4 \times 10^3 \sqrt{\frac{T_\perp}{KeV}} \sqrt{\frac{n_e}{cm^{-3}}} \; \frac{\widetilde{W_o}^{3/2}}{(1+\widetilde{W_o})^{-\frac{1}{4}} + \frac{1}{3}\widetilde{W_o}(1+\widetilde{W_o})^{-\frac{3}{4}}} \qquad (8)$$

$$v_g = 0. \qquad (9)$$

Numerical analysis of this set of equations (1-9) permits to analyze the Weibel instability due to the laser pulse field in the laser fusion plasma corona.

The excited Weibel modes by this mechanism are not convective. That, $v_g \sim f_1 = 0$. But other Weibel sources, such as due to the gradient of temperature and density, in the corona, can participate to the convection of these modes. We have presented on the, FIG.2., the Weibel instability growth rate spectrum $\gamma(k\lambda)$, where k is the Weibel mode wave number and λ is the electron mean free path. We have also presented, on the FIG.3., the γ_{max} as a function of the scaling parameter, $\widetilde{W_o}$. It has been shown that the unstable Weibel modes are non collisionals, $k\lambda > 1$. The growth rate of the most unstable Weibel mode, γ_{max}, is $\gtrsim 10^{11}s^{-1}$ (eq. 8) in the vicinity of the critical layer, $\omega_L = \omega_p$. The growth rate calculated in this model gives the results of the Fokker-Planck simulation especially for low values of the scaling factor, $\widetilde{W_o}$.

4. Scaling laws

In the inertial target fusion experiments, the created plasma parameters are interconnected to the parameters of the incident laser pulse to the target. In the reference [12], a scaling law of the electron temperature is established. This law is obtained by the compute of the energy stock in the critical layer, so:

$$\frac{T_{ec}}{KeV} = 4{,}3\left(\frac{I_a}{10^{14}W/cm^2}\right)^{2/3}\left(\frac{\lambda_L}{\mu m}\right)^{4/3}. \qquad (10)$$

The critical density is then given by:

$$\frac{n_c}{cm^{-3}} = 1.1 \times 0^{21}\left(\frac{\lambda_L}{\mu m}\right)^{-2}. \qquad (11)$$

The collisions frequency is given as a function of the electronic corona density and temperature:

$$\frac{\vartheta_c}{s^{-1}} = 3.4 \times 10^{-9}(Z+1)\frac{n_e}{cm^{-3}}\left(\frac{T_e}{KeV}\right)^{-3/2} ln\Lambda. \qquad (12)$$

lnL means the Coulomb Logarithm. In the case of the laser fusion plasma $lnL \approx 10$.
By taking into account the equations (10, 11), the collision frequency in the critical layer for an isothermal corona can be expressed as a function of the laser parameter as:

$$\frac{\vartheta_c}{s^{-1}} \approx 4.2\times 10^{11}(Z+1)\left(\frac{I_a}{10^{14}W/cm^2}\right)^{-1}\left(\frac{\lambda_L}{\mu m}\right)^{-4}. \tag{13}$$

$$\frac{\vartheta_c}{\omega_L} \approx 2.2\times 10^{-5}(Z+1)\left(\frac{I_a}{\frac{10^{14}W}{cm^2}}\right)^{-1}\left(\frac{\lambda_L}{\mu m}\right)^{-3}. \tag{14}$$

This equation shows that the collisions are efficient as the laser wave length, λ_L, is shorter.

The scaling parameter (eq. 2) is given by:

$$\widetilde{W}_o = 5.4\times 10^{-4}\left(\frac{I}{10^{14}W/cm^2}\right)\left(\frac{I_a}{10^{14}W/cm^2}\right)^{-2/3}\left(\frac{\lambda_L}{\mu m}\right)^{2/3}\times$$

$$\left[1-2.5\times 10^{-10}(Z+1)^2\left(\frac{I_a}{10^{14}W/cm^2}\right)^{-2}\left(\frac{\lambda_L}{\mu m}\right)^{-6}\right]. \tag{15}$$

$I_a = AI_L$, where I_L is the laser pulse intensity and A is the absorption coefficient.
The absorption coefficient [1,8] due to the inverse bremsstrahlung mechanism for a linear density spatial profile, is obtained in the WKB approximation by:

$$A_{IB} = 1-\exp\left(-\frac{32}{15}\frac{v_t}{c}\frac{L_n}{\lambda}\right). \tag{16}$$

$L_n = \left|\frac{\vec{\nabla}n_e}{n_e}\right|^{-1}$ is the density gradient length.
In the case where we can consider a mono dimensional corona expansion, the density gradient length is proportional to the expansion velocity, C_s, and to the laser pulse duration, τ, so:
$L_n \approx C_s\tau$, where $C_s = \sqrt{\frac{ZT_i}{m_i}}$ is the sound speed and τ is the laser pulse duration. $Z, T_i\ and\ m_i$ mean respectively the plasma ionization number, the ions temperature and the ion mass.
Using eq. (10) L_n can be given by the following scale law:

$$\frac{L_n}{\mu m} \approx 9.4\times 10^{11}\left(\frac{I_a}{10^{14}W/cm^2}\right)^{1/3}\left(\frac{\lambda_L}{\mu m}\right)^{2/3}\frac{\tau}{s}. \tag{17}$$

The inverse bremsstrahlung absorption is then expressed as:

$$A_{IB} = 1-\exp\left(-5.51\times 10^{2}\left(\frac{I_a}{10^{14}W/cm^2}\right)^{2/3}\left(\frac{\lambda_L}{\mu m}\right)^{4/3}\frac{\tau}{ns}.\right). \tag{18}$$

Other mechanisms participate to the absorption of the laser energy in the corona; namely the resonance absorption mechanism in the vicinity of the critical layer. Then, the effective absorption be greater than that due to the inverse bremsstrahlung mechanism (eq. 18), $A > A_{IB}$. In the laser fusion experiments using nanosecond laser pulses with laser wave length $\lambda_L < \mu m$, we can consider in a good approximation that the laser pulse energy is totally absorbed: $I_a = I_L$ and $A = 1$.
It is important to express the local laser intensity, I, as a function of the laser pulse parameters. The spatial evolution in the corona of the laser electric field magnitude, $E_0(x)$ in the case of a linear density profile is given by the Ai Airy function [1,3,8], so:

$$E_0(x) = 2\sqrt{\pi}\left(\frac{\omega_L L_n}{c}\right)^{\frac{1}{6}} E_v \mathrm{Ai}(\xi)\exp\left(-\frac{\vartheta_c(x)}{\omega_L}\right), \qquad (19)$$

where E_v is the laser electric field magnitude in the interface vacuum-plasma, $(x = L_n)$, and $\xi = \left(\frac{\omega_L L_n}{c}\right)^{\frac{1}{6}}\left(\frac{x}{L_n} + i\frac{\vartheta_c(x)}{\omega_L}\right)$ is a spatial dimensionless coordinate.
The laser intensity in the critical layer ,$(x = 0)$, is given by:

$$I_c = 4\pi\left(\frac{\omega_L L_n}{c}\right)^{\frac{1}{3}} I_v \mathrm{Ai}^2(0)\exp\left(-\frac{2\vartheta_c(0)}{\omega_L}\right), \qquad (20)$$

By taking into account the equations (10,11,13,17), the laser intensity in the critical layer can be expressed as:

$$\frac{I_c}{\frac{10^{14}W}{cm^2}} = 2.8\times10^4 \frac{I_L}{\frac{10^{14}W}{cm^2}}\left(\frac{I_a}{\frac{10^{14}W}{cm^2}}\right)^{1/9}\left(\frac{\lambda_L}{\mu m}\right)^{-1/9}\left(\frac{\tau}{s}\right)^{1/3}\times$$

$$\exp\left(-4.4\times10^{-5}(Z+1)\left(\frac{I_a}{\frac{10^{14}W}{cm^2}}\right)^{-1}\left(\frac{\lambda_L}{\mu m}\right)^{-3}\right). \qquad (21)$$

The equations (10,11,15,21) allow us to establish a scaling law for the growth rate of the most unstable Weibel mode in the critical layer as:

$$\gamma_{max} = 0.18\left(2 - A - 2\sqrt{1-A}\right)^{\frac{3}{2}} A^{-\frac{1}{2}} I_L \lambda_L^{1/2} \times$$

$$\exp\left[-6.6\times10^{-5}(Z+1)A^{-1}{I_L}^{-1}\lambda_L^{-3}\right]. \qquad (22)$$

Here I_L is the laser pulse intensity and A is the absorption coefficient. Note that I_L in W/cm^2, λ_L in μm and γ_{max} in $10^{11}s^{-1}$.
In the case of the total absorption, $A = 1$, the above expression is simplified as:

$$\gamma_{max} = 0.18 I_L \lambda_L^{\frac{1}{2}} \exp\left[-6.6\times10^{-5}(Z+1){I_L}^{-1}\lambda_L^{-3}\right]. \qquad (23)$$

We point out from this that $\gamma_{max} \sim I_L \lambda_L^{\frac{1}{2}}$. This corresponds to the results of the reference [2] using the Fokker-Planck theory.
This expression permits to optimize the laser parameters using in the fusion experiments in order to minimize the energy losses due to the Weibel instability.

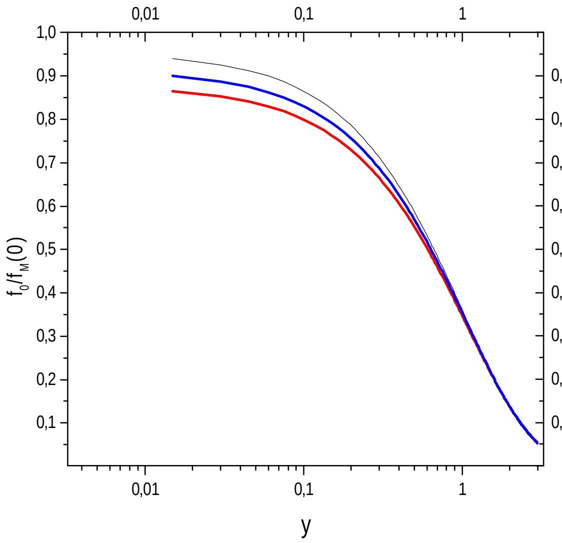

FIG.1. Isotropic distribution function as function of y for several values of the scaling parameter $\widetilde{W}$.
The red curve corresponds to $\widetilde{W}_o = 0.01$, the blue curve corresponds to $\widetilde{W}_o = 0.02$ and the black one corresponds to $\widetilde{W}_o = 0.03$

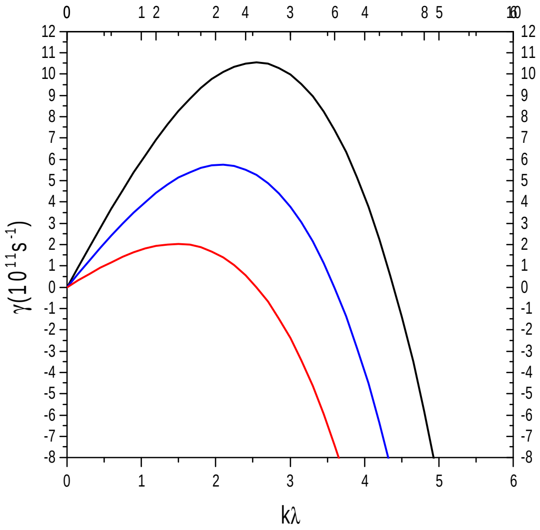

FIG.2. Weibel instability growth rate spectrum: $\gamma(k\lambda)$.
k is the wave number of the Weibel mode and λ is the mean free path of the electron.
The red curve corresponds to $\widetilde{W}_o = 0.01$, the blue one corresponds to $\widetilde{W}_o = 0.02$ and the black one corresponds to $\widetilde{W}_o = 0.03$.
$T = 1KeV$ *and* $n_e = 10^{21} cm^{-3}$

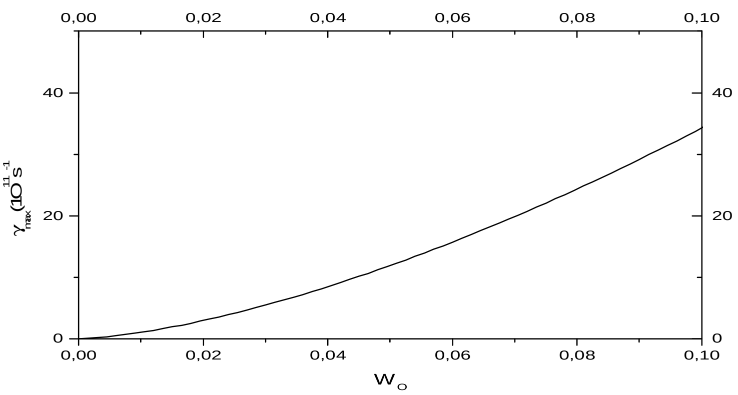

FIG. 3. γ_{max} as function of the scaling parameter $\widetilde{W}_o$.
$T_e = 1\ KeV$ *and* $n_e = 10^{21} cm^{-3}$

5. Conclusion

In the present work, the Weibel instability is studied in the corona of the laser fusion plasma through a theoretical model. The unperturbed electronic distribution function is supposed to be a local bi-Maxwellian. It has been shown Weibel modes highly unstable in the vicinity of the critical layer : $\gamma_{max} \gtrsim 10^{11} s^{-1}$. The γ_{max} is proportional to $I_L \lambda_{\mathrm{L}}^{\frac{1}{2}}$. Practical scaling laws are established for the instability parameters. The results of this paper are in good agreement with the results of the reference [2] founded in the frame of the kinetic Fokker-Planck theory. We assume that the theoretical study presented on this paper permits to optimize the laser pulse parameters in order to have minimum energy losses in the laser fusion experiments.

6. References

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