Reversed-Field Pinch Contributions to Resistive Wall Mode Physics and Control

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Abstract: Optimal feedback control of resistive-wall modes (RWM) is of common interest for toroidal fusion concepts that use conducting walls for stabilization of ideal MHD modes. From the RWM control point of view, the RFP situation is in many respects similar to the advanced tokamak situation in the presence of very low plasma rotation, where the most effective stabilizing mechanism is the feedback action of a set of active coils. Results from EXTRAP T2R (Sweden) and RFX-mod (Italy) RFP experiments have shown that full feedback control of multiple RWMs is possible and their deleterious effects can be completely suppressed. However it is now important to optimize the RWM control systems both for the RFP and tokamak configuration for future implementation. Important aspects of optimization are effective mode identification and tracking capability, avoidance of the harmful effects of sideband modes (aliasing) in the control spectrum, minimized power requirements and robust controller stability. The paper describes collaborative work carried out on the two RFP experiments. Controller models based on the mode harmonic control concept and on a state-space multiple-input multiple-output intelligent shell concept are studied. Progress in development of optimal control schemes are presented both through experimental studies and simulations.

1. Introduction

Optimal feedback control of resistive-wall modes (RWM) is of common interest for toroidal fusion concepts that use conducting walls for stabilization of ideal MHD modes characterised by field harmonics, $b_{m,n} \propto \exp(im\vartheta + in\phi)$, where *m* and *n* are the poloidal and toroidal mode numbers [1]. A RWM radial field perturbation characterised by (*m,n*) measured at the wall has a growth rate, $\gamma_{m,n}$, and the dynamics is described by $\tau_{m,n}\dot{b}_{m,n} - \tau_{m,n}\gamma_{m,n}b_{m,n} = b^{ext}_{m,n}$, where $\tau_{m,n}$ is the wall penetration time for the mode and $b^{ext}_{m,n}$ is that part of the resonant perturbation measured at the wall by the sensor coils that is produced by sources external to the plasma (active coils and field errors). For the advanced tokamak, ideal kink modes may be destabilised by pressure gradients [2]. For the reversed-field pinch (RFP), it is necessary to simultaneously control a finite range of current-density gradient-driven unstable ideal kink modes [3]. From the RWM control point of view, the RFP situation is in many respects similar to the advanced tokamak situation in the presence of very low plasma rotation, where the most effective stabilizing mechanism is the feedback action of a set of active coils.

RWM control systems incorporate arrays of magnetic sensor coils and active saddle coils (actuators) distributed around the torus wall (or "shell"). Feedback is implemented by a real-time controller which uses, in an optimal way, the admissible actuators for MHD control in the form of currents in the active coils to constrain the MHD mode evolution at a specified reference spectrum by responding to measured sensor voltages. A digital controller has been developed by Consorzio RFX and implemented in a collaborative effort on both the RFX-

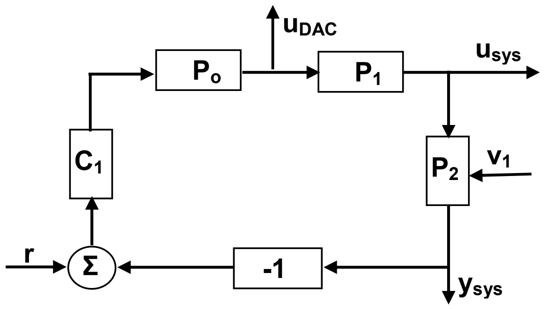

FIG. 1. Schematic of the signal routing closing the feedback loop. r(t) is a reference; C_1 is the feedback control function; $u_{DAC}(t)$ represents the ideal active coil current; P_0, P_1 represents a time delay and a composite active coil and power amplifier, respectively. $u_{sys}(t)$ represents the active coil control output current; P_2 represents the front-end RWM wall/plasma dynamics; $v_1(t)$ is an exogenous signal representing the field errors; $y_{sys}(t)$ represent the time-integrated sensor voltages.

mod experiment (Italy) [4,5] and the EXTRAP T2R experiment (Sweden) [6,7]. Typical routing of the signals in the closed control loop is shown in FIG. 1. The intelligent shell (IS) idea, proposed by Bishop [8], can be implemented as a set of decentralized single-input single-output (SISO) PID (proportional-integral-differential) controllers where each controller channel is dedicated to a simplified signal routing coupling a single sensor output (measure of net magnetic flux linking the sensor loop) to the current input of the spatially coincident active coil needed to zero the flux.

The IS control system has been implemented and optimized with the result that substantial improvement in RFP plasma discharges has been achieved with IS PID controllers. Results from EXTRAP T2R [7] have shown that full feedback control of multiple RWMs is possible and their deleterious effects can be completely suppressed. In FIG. 2 the plasma current and the radial field perturbation penetrating the wall is shown for discharges with and without active IS RWM control. Without active control, the perturbation grows on the wall penetration time scale (τ_o is 6 ms for T2R), and the discharge terminates after 18 ms. With active control, the radial field at the sensors is suppressed and the pulse length is 95 ms, limited by the discharge power supplies.

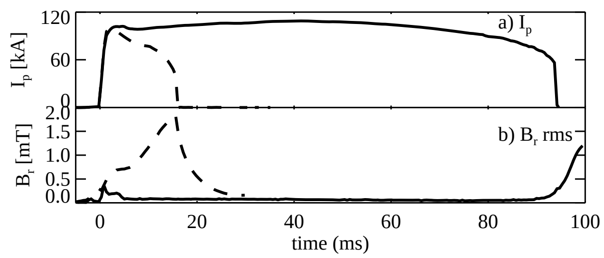

FIG.2. Discharge current and RWM radial field perturbation for two T2R discharges. The solid curve is with active RWM control and the dashed curve is without active control.

A related control system concept is based on harmonic mode control where discrete Fourier transform (DFT) harmonics are calculated in real time from the sensor coil array measurements by the digital controller and feedback is performed by harmonics produced by the active coils. The corresponding set of PID control functions therefore addresses the MHD control dynamics in the conventional harmonic form for MHD stability analysis. The intention of both the IS and mode control (MC) concepts is via feedback to provide a virtual shell (VS) boundary simulating an ideal wall. The MC system has successfully stabilised the multiple RWMs in RFX-mod [5]. Also, the flexibility of the control systems working in the Fourier space allows for dedicated experiments on RWM physics which can be compared with MHD theory. An example is selective control of one or more RWMs in order to study their mutual interactions and their interactions with the bulk plasma or with other MHD instabilities. Under these conditions very precise growth rate measurements are performed, with the additional aim of contributing to the continuous refinement of numerical tools common to both RFPs and tokamaks. The flexibility of mode control is also shown in FIG. 3 where the growth rate of a selected

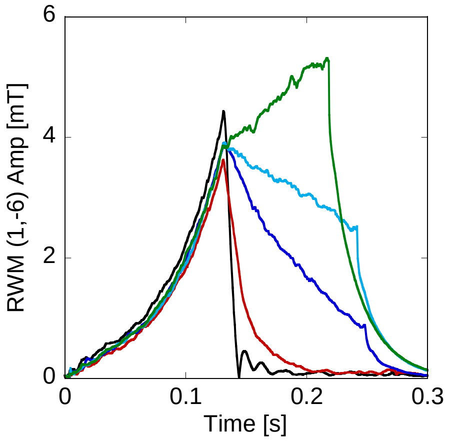

FIG. 3. Proportional gain scan on the (1,-6) RWM. Increasing gain values follow the colors from green (lower gain) to black (higher gain).

unstable RWM is modified by using a range of different proportional gains in order to simulate the effect of control systems with different power capabilities.

While demonstration of the active stabilisation of RWMs in the RFP is a noteworthy milestone, it is important to optimize the RWM control systems for future implementation both for the RFP and tokamak configurations. The paper describes collaborative work carried out on the two European RFP machines. Experimental studies of RWM control physics and progress in development of optimal control schemes are presented. In addition, simulation modelling of the functioning control systems is presented. Developments of advanced control models are discussed, with particular stress on co-ordinated efforts with tokamak MHD control studies.

2. Generic aspects of RWM active control

There are a number of important areas where the development of active RWM control systems for the RFP and the tokamak concepts face common issues:

Implementation of advanced control theory.

- Effective mode identification and tracking capability and avoidance of the harmful effects of sideband modes (aliasing) in the control spectrum.
- Fast transient response time, minimized power requirements and robust controller stability.

Plant modelling.

- Development of a robust plant model including features of the plasma, wall, sensors, actuators, field error environment, noise environment and controller dynamics.
- Incorporation of 3D effects due to a non-axisymmetric wall.

Development of the MHD physics understanding relevant for control.

- Studies of RWM stability growth rate dependence on rotation.

Substantial advances have been made in these areas and an overview of the progress is presented in the following sections.

3. Implementation of advanced control theory

Although the mode dynamics are linear for each mode and the ideal modes are not coupled, the dynamics in the control model are not spatially localised. Therefore the strategy of local response to local perturbation implemented by a SISO controller set is not optimal. One complication is mode aliasing. The active coils have a discrete size and the field produced has a spectrum of high n sideband harmonics, whose aliasing pollutes the sensor measurements. The sampling theorem states that the DFT harmonics derived from the array of sensor coils correspond to the Fourier harmonics only if aliasing does not occur. The zeroing of the sum of $b_r^{m,n}$, which is the content of the sensor measurement, does not imply that the Fourier harmonics are zero. The sideband harmonics are produced by the control coils or arise from edge field errors that are enhanced by resonant field amplification [9]. This generic problem is ubiquitous and must be addressed for both the RFP and tokamak configurations. Two approaches have been pursued to correct this sideband aliasing problem.

In RFX-mod, the MC controller has been improved to remove the aliasing effects. The geometry and parameters of the wall and of the active coil and sensor coil systems are known. The sideband aliasing due to the currents flowing in the active coils is calculated in real time and subtracted from the sensor signal measurements so that the feedback variables are the Fourier harmonics and not the aliased measurements. This clean mode control (CMC) concept has been successfully tested on the RFX-mod device and is now part of standard operation [10].

In T2R, the IS controller concept has been developed in two main directions: *i)* by formulating a multiple-input-multiple-output (MIMO) model based on theory [11], and *ii)* by pursuing a mainly experimental process control approach [12]. The system dynamics for the MIMO model (location P_2 in FIG. 1) are expressed in state-space form as

$$
\begin{aligned}
(d/dt)\mathbf{x} &= A\mathbf{x} + B\mathbf{u}_{sys} + N\mathbf{v}_1 \\
\mathbf{z} &= M\mathbf{x} \\
\mathbf{y}_{sys} &= C\mathbf{x} + \mathbf{v}_2
\end{aligned}
\tag{1}
$$

where $\mathbf{u}_{sys}$, $\mathbf{v}_1$ and $\mathbf{y}$ are defined in FIG. 1, $\mathbf{x}$ is a vector of MHD modes, i.e. the *state* of the MHD fluid. The system matrices *A, B* and *C* are structurally defined by the parameters and geometry of the wall, actuators and sensors. Here $\mathbf{z}$ is an optional vector expressing the desired performance, and *M,* which is a key factor for implementing process control, relates the MHD harmonics $\mathbf{x}$ to the merit vector $\mathbf{z}$. It is this merit $\mathbf{z}$ that, in principle, defines the control system objective. Given measurements $\mathbf{y}_{sys}$, we need an observer of $\mathbf{z}$, presumably a subset of *important* MHD harmonics, yielding the estimate $\{\hat{\mathbf{z}}\}$, which is the inferred (model-based filtering, e.g. *Kalman*-filter) multivariable signal the control system reads. The source term $N\mathbf{v}_1$ is another issue that requires improved modelling, as it bundles effects of field-errors and MHD noise. Also, $\mathbf{v}_2$ is a white noise signal.

The potential for improvement of the control system by incorporating the MIMO model, Eq. (1), includes:

- With the state-space model in hand, the controller can be optimized using advanced control theory methods.
- The aliasing is, by definition, included in output-relation $\mathbf{y}_{sys}=C\mathbf{x}$ and therefore controller synthesis automatically handles the aliasing effect.
- The inherent external field error state vector in $\mathbf{v}_1$ can be estimated, e.g. [11].
- The IS concept can be generalised to enable output tracking capable of sustaining MHD modes at preselected (non-zero) amplitudes and phases [12].

A numerical simulation illustrating the ideas is shown in FIG. 4. The parameters of T2R are used; the active coil and sensor coil arrays are 4 (poloidal) by 32 (toroidal) [7]. The coils are up-down and inboard-outboard series connected (i.e. m=1, 3,…) so that there are 64 control channels and 64 sensor channels. The objective of the controller is to keep a sub-band that contains the unstable RWM harmonics, (m=1, n=-16..+15) of modes $\mathbf{z}$ at a reference $\mathbf{r}$, $\mathbf{r}\sim\mathbf{z}$. Aliasing effects are seen for higher-harmonics, but the control system, can theoretically keep track of these using its internal model.

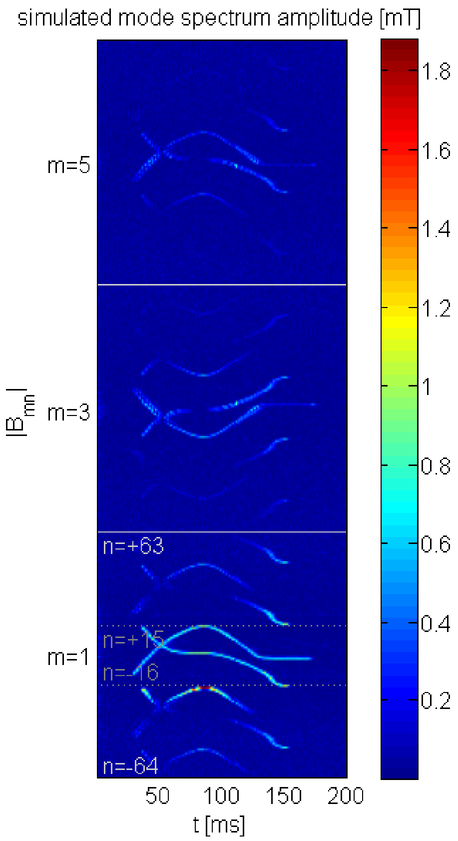

FIG. 4. Simulated mode spectrum amplitude (mT) generated by the MIMO controller when a spectrum sweep reference was applied for a 200-ms fictitious shot.

Modelling has also been used to optimize the baseline IS PID gains by particularly focusing on the implications of control system latency effects. The predicted optimal gains *(Kp,Ki,Kd)*, obtained through closed-loop eigenvalue minimization of a delay differential equation (DDE), were implemented in the controller and tested on the T2R device, which resulted in improved PID controller performance [11] compared to the gains that had been empirically determined [13]. Improvements were in the sense of reduced magnetic field energy at the sensors, admittedly at the cost of higher control power requirements.

4. Development of plant modelling

Salient features of the plant are the plasma stability dynamics including possible damping effects due to plasma rotation, the model of the wall used to calculate the transient fields at the wall and knowledge of magnetic field errors. In the early studies carried out on RFX-mod and T2R, ideal MHD linear dynamics has been successfully used to model the plasma stability when the RWM is essentially stationary in the laboratory frame and a smooth wall is used to estimate field penetration dynamics at the wall. Concerning the wall model, optimal mode control for both the RFP and the tokamak requires consideration of three-dimensional (3D) effects on the wall penetration time and associated mode growth rate. Also, the transient spectra of control harmonics produced by the currents in the active coils, calculated using the geometry and parameters of the coils, contain error harmonics due to the 3D wall. Magnetic field errors with external sources are also present. Here again the most difficult issue is the modelling of the 3D wall. It can also be pointed out that in addition to externally produced field errors and the field errors in the control fields, transient axisymmetric plasma equilibrium changes introduce field errors due to image currents in the non-axisymmetric walls.

An understanding of the effects of plasma rotation on RWM growth rates and benchmarking of the effects of the complex 3D conducting structures surrounding the plasma on RWM growth rates are urgent issues in the modelling of RWM control in toroidal devices and in particular in ITER. Since the RWM stability in the RFP configuration is not affected by dissipation and flow, the modelling of the RWM can focus on the analysis of 3D effects removing other sources of uncertainties. To this purpose, a collaboration between the RFX-mod, UKAEA, and CREATE groups implemented a 3D model of the RFX-mod device in a new computational tool called CarMa [14]. CarMa can rigorously take into account the three-dimensional details of the conducting structures (see FIG. 5) in the solution of the plasma stability problem, and can also account for multiple toroidal Fourier harmonics in the plasma response, which is mandatory in RFP. In this way CarMa predictions have been benchmarked against experimental data for the first time, which is a very useful point for its applications to ITER. The CarMa RWM growth rates have been compared with RFX-mod data, and, for further comparison, with the predictions given by the cylindrical code ETAW and by the toroidal MHD code MARS-F [15]. Some of the results are presented in Table I and show generally a good agreement between experiment and numerical models, with a clear improvement when 3D effects are taken into account. This demonstrates that the inclusion of

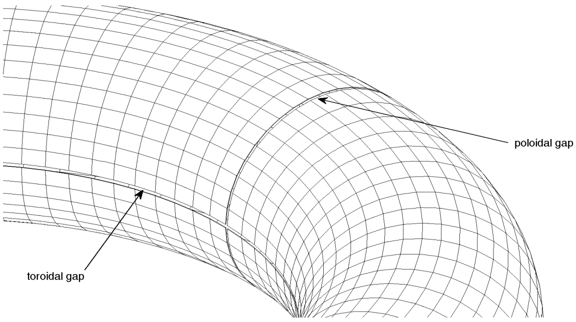

FIG.5. Shell geometry and gaps as described in the mesh used in CarMa simulations.

TABLE I: Comparison between numerical and experimental RWM growth rates.

	ETAW	MARSF	CarMa	Exp.
n=4	5.27	5.07	7.30 7.48	≈6
n=5	8.63	8.55	12.8 13.1	≈12
n=6	14.5	14.4	22.6 23.4	≈22

3D boundary effects improves the agreement with experimental data and that three-dimensional electromagnetic effects may play a significant role in RWM stability and has a direct implication to ITER predictions. Plans are present to extend in the near future the experimental database to allow a comparison also on modes with lower toroidal number where the effects of machine structure details might play an even more important role. In addition, the possibility in RFX-mod to distinguish the control of odd and even poloidal mode numbers (mainly m=0 and m=1 for the RFP configuration) by independently feeding and controlling each of the four active coils of a poloidal cross section will give the opportunity for a careful comparison of the poloidal sideband structure of RWM as measured in the experiment and as simulated by CarMa.

The MIMO model developed for EXTRAP T2R also provides opportunities to address the field error problem, due to imperfections in the plant such as misalignment of the torus and global magnetic coil systems, coil current feeds and cuts and ports in the wall. The error fields are present in the sensor measurements and ideally the corresponding harmonics should also be suppressed at the plasma edge by the control system. The field errors could be included in the control model in order to optimize the control system. In the state-space model, Eqs. (1), a possible external field error state vector augmentation can be formed from $\mathbf{v}_1$. This error-signal plausibly can be split into a vessel *signature* part, essentially being deterministic, and a (*colored*) noise part. The harmonics of these external field errors affect the linear mode dynamics. These errors present during a typical open-loop plasma discharge have been estimated via model-based filtering, such as a Kalman Filter [11], and developments in this direction are pending. These general issues are evidently very relevant to tokamak control.

5. Development of the MHD physics understanding relevant for control.

The main goal of the RWM control studies is to develop optimal control systems for resistive-wall modes that are adequate to allow operation of steady state RFP plasmas and to allow advanced tokamak operation. However MHD mode control also opens the possibility for general studies of MHD physics in the RFP configuration, for example the study of mode locking, the dynamo effect and the control of quasi-single helicity states [16]. Optimized

RWM control schemes can address RWM physics issues which in turn can lead to an improved comprehension of the mechanisms leading to RWM instability in the RFP configuration. An example are the ongoing studies carried out in RFX-mod in a collaboration between Consorzio RFX and the ASDEX-Upgrade group on the controlled rotation of a RWM mode with the aim of a controlled investigation of the effects of rotation on its growth rate [17]. The subject is very relevant for the tokamak configuration, in particular when RWM stabilization in ITER is considered. In RFPs the problem is complicated by the fact that RWMs grow as non-resonant instabilities that appear to be wall locked from discharge start. Their characteristics prevent the application of the torque mechanisms generally used in the tokamak case when modelling similar experiments. The flexibility of the control systems working in the Fourier space (Mode Control) allowed for dedicated experiments using in particular an evolution of this control, i.e. the feedback application of complex gains on selected unstable RWMs. Experimental results obtained in RFX-mod are shown in FIG. 6 and indicate that finite amplitude RWMs can be put in rotation by the action of the external control system with a phase velocity that depends only on the phase difference between the mode and the external perturbation. The mode amplitude instead is kept under control by the real part of the proportional gain. The modelling of experimental results suggests that in RFPs a pure electro-magnetic description is sufficient to describe the growth rate evolution and confirm that relative rotation velocities between the mode and the plasma much lower than the Alfvén one do not affect RWM stability in the RFP configuration, as already predicted numerically [18].

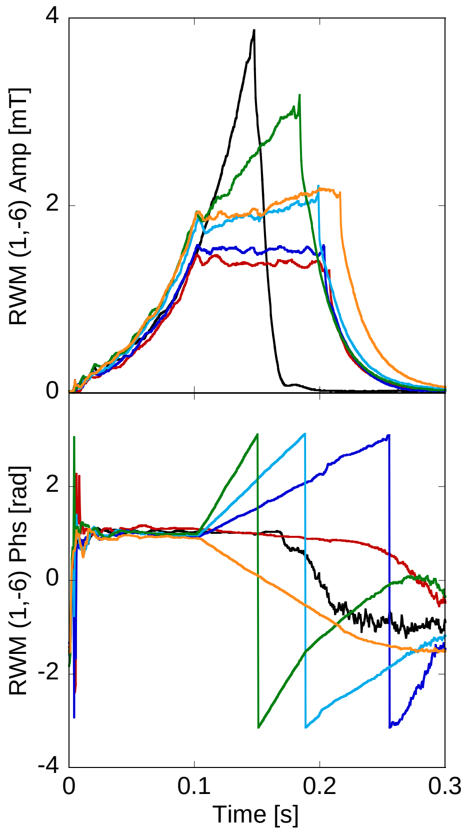

FIG. 6. Imaginary gain scan on the (1,-6) RWM in RFX-mod. RWM rotation can be induced in both directions in a fully controlled way.

In a collaborative effort, the same experiments have been recently replicated on the T2R RFP in order to highlight the effect of different boundary conditions on RWM rotations induced by the active control system. Using the same experimental procedure, it was possible to unlock the selected RWM in T2R as well. Experimental data clearly confirm that for small angles between the plasma and the external mode ($\Delta\phi$<45°), the rotation velocity is controlled by the magnitude of the imaginary part of the gain, while the residual mode amplitude depends only on the magnitude of the real part of the gain. However, preliminary results seem to suggest that in T2R in case of faster rotations ($\Delta\phi$>45°), the RWM amplitude can decrease also as result of the induced rotation itself, without any change in the real part of the gain. Preliminary data show that slow RWM rotation is obtained, similar to the RFX results.

6. Summary

In conclusion, the present generation of RFP experiments, with the help of state-of-the-art active control systems and numerical models, can explore new MHD control scenarios and actively complement similar research on other toroidal confinement devices. For implementation of RWM control systems in future experiments, both for the RFP and tokamak configurations, it is important to optimize the RWM control system and the ongoing studies focus on issues that are important for this optimization.

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