The role of stochastization in fast MHD phenomena on ASDEX Upgrade

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Abstract. Studies of fast MHD events in ASDEX-U suggest that stochastization plays an important role in these processes. In spite of the short time duration and small region of stochastization, it can lead to strong changes in plasma confinement. Main reason for such influence is a strong mixing of the magnetic field lines which destroy magnetic surfaces and strongly increase radial transport. In this paper three such phenomena are discussed: frequently interrupted regime of neoclassical tearing mode (FIR-NTM), minor disruption due to interaction of the (2,1) and (3,1) tearing modes and sawtooth crash. The role of stochastization of magnetic field lines is analyzed by applying the mapping technique to trace the field lines of toroidally confined plasma. The sawtooth crash dynamics is also studied by means of spectral analysis and reconstruction of phase trajectories using delay coordinates which are standard techniques for stochastic systems and have been employed for identification of the transition to chaos in different physical systems.

1. Introduction

Magnetic equilibrium in tokamaks can be represented as a set of nested magnetic surfaces. MHD instabilities deform these surfaces in case of an ideal MHD mode and create island structures in case of a resistive mode. These modes are typically rotating with different rotation frequencies which screen perturbations from each other. Such a screening effect vanishes when perturbations are coupled. During this coupling stage, a short stochastic phase can appear. The main characteristics of this phase is a strong mixing of the magnetic field lines which destroy closed magnetic surfaces and strongly increase the radial transport. In spite of the short time duration and small region of stochastization, it can lead to very strong changes in plasma confinement. Studies of fast MHD processes in ASDEX-Upgrade suggest that indeed the stochastization plays an important role in these events. In this paper three such phenomena are discussed: frequently interrupted regime of neoclassical tearing mode (FIR-NTM), minor disruption due to interaction of the (2,1) and (3,1) tearing modes and the sawtooth crash.

The role of stochastization of magnetic field lines is analyzed by applying the mapping technique to trace the field lines of toroidally confined plasma. In this method [1] magnetic field lines are regarded as trajectories of the Hamiltonian system. Practical implementation of this technique requires knowledge of the safety factor and of the MHD perturbations. Determination of the shape and amplitude of the MHD perturbations is a challenging task, because of the large uncertainties in the measurements. We have used combinations of all main MHD diagnostics (magnetic measurements, ECE, Soft X-ray cameras) to deduce these perturbations from experimental data and convert them into the form suitable for the Hamiltonian formalism [2, 3, 4].

The important parameters for the creation of a stochastic region are:

(i) amplitudes of perturbations;
(ii) safety factor profile;
(iii) number of perturbations with different helicities;
(iv) coupling of perturbations.

It would be shown that each of these conditions is important for creation of a stochastic region in tokamaks.

2. Frequently Interrupted Regime of Neoclassical tearing mode (FIR-NTM)

During the FIR-NTM the amplitude of the NTM after reaching a certain size suddenly drops to a much smaller value [5]. After this the mode growth starts again. In this way the NTM amplitude never reaches its saturated value. The time in which these amplitude drops occur is very short (about 500 μs), much shorter than the resistive MHD reconnection rate (few 10s of milliseconds in the ASDEX Upgrade).

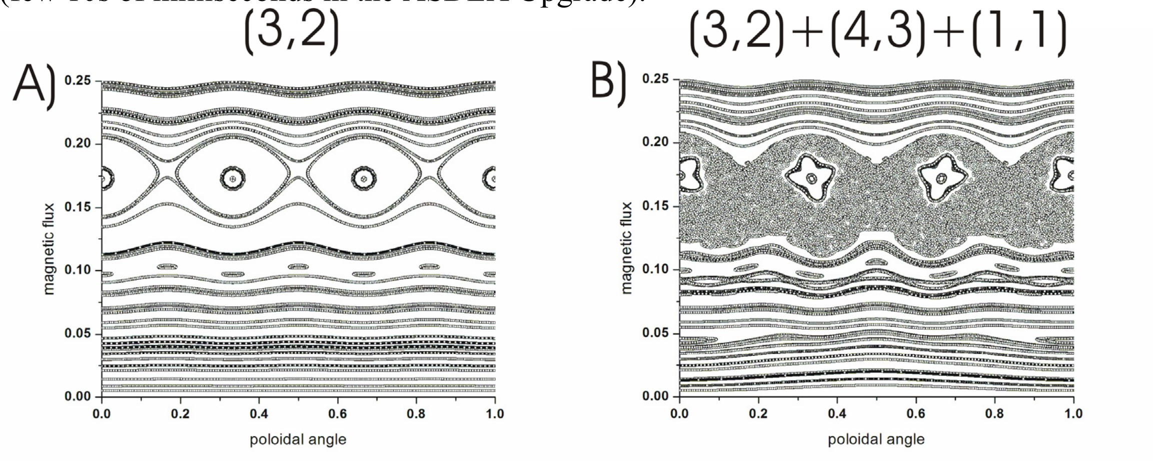

Fig.1. Poincare plots for single (3,2) tearing mode (A) and for interaction of (3,2) tearing mode, (4,3) ideal mode and (1,1) ideal modes (B). Stochastic region is clearly seen in figure (B).

It was shown that this experimental observation can be explained by stochastization of magnetic field lines when the island separatrix is destroyed [3] (see figure 1.). We have found that experimental amplitudes of the perturbations are always sufficient to stochastise the magnetic field (condition (i)), but stochastization appears only during the coupling of the modes (condition (iv)). The (1,1) mode, which is needed for a nonlinear coupling between the modes, has a negligible influence on stochastization itself. In this example, stochastization plays a positive role and reduces influence of the NTM on the plasma confinement. It is interesting that similar result was found in non-linear MHD simulations [6]. The time of the fast drop can be also explained by stochastisity if these experimental perturbation are used for diffusion calculations [7].

3. Minor disruption due to the interaction of the (3,1) and (2,1) tearing modes

It was observed in ASDEX Upgrade discharge that series of minor disruptions are accompanied by the interaction of the (3,1) and (2,1) modes [8]. Such a minor disruption leads to temporary deterioration of confinement and flattening of the temperature profile. We have modelled this disruption by using the perturbation amplitude obtained by means of ECE measurements. In this case modes are always coupled (condition (iv)) but stochastization between the resonant surfaces appears only if the amplitude of the (2,1) is higher than a threshold value [3]. In this example, stochastization plays a negative role and leads to a minor disruption. This example together with FIR-NTM case demonstrate importance of the all conditions for creation of the stochastisity discussed above.

4. Sawtooth crash

Investigation of sawtooth crashes in ASDEX Upgrade shows that in many cases the magnetic reconnection is not complete [9], which means that all complete reconnection models are in contradiction with experimental observations. It was shown by means of the mapping technique that amplitudes of the primary (1,1) mode together with its harmonics are sufficient to stochastize the region if the central q is less than 0.85-0.9. This is in good agreement with measurements of the safety factor profile and allows one to explain the existence of the mode after the sawtooth collapse [4]. Poincare plots are shown together with the corresponding safety factor profiles in figure 2.

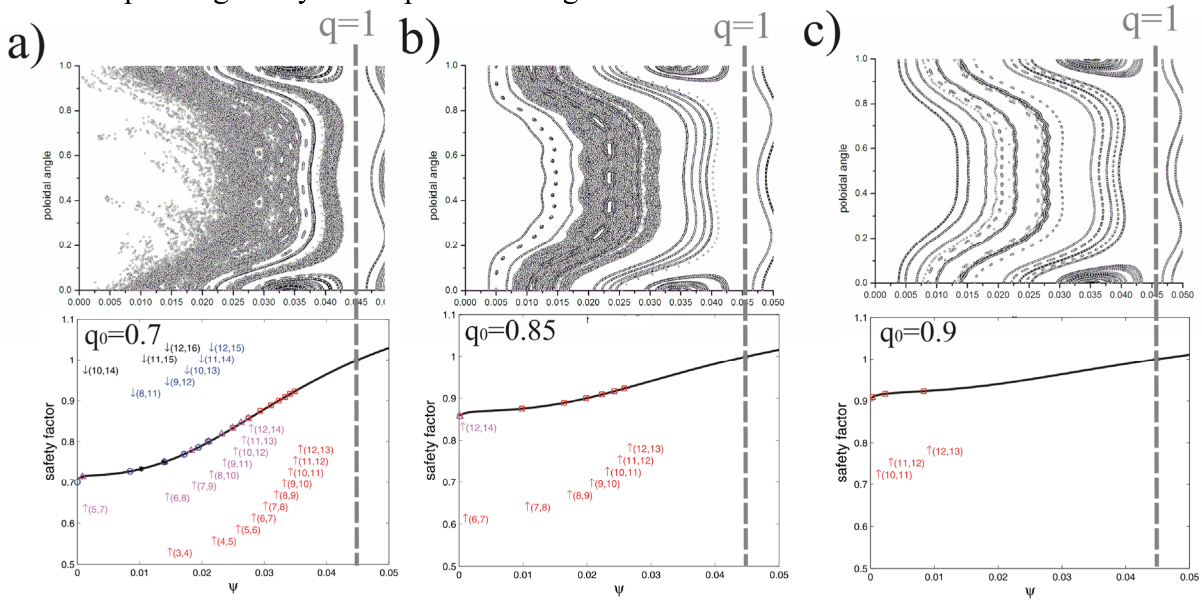

Fig. 2. Poincare plots based on reconstructed experimental perturbations due to primary (1,1) mode and its harmonics (2,2) and (3,3) are shown together with corresponding safety factor profiles. All calculations are made with the same perturbations and different safety factor profiles. Note that stochastization strongly depends on the existence of the low-order rational surfaces which are marked on safety factor curves. a) central q-value is 0.7; b) central q-value is 0.85; c) central q-value is 0.9

The low-order rational surfaces are marked on safety factor curves. It can be clearly seen that with increasing q_0 stochastic region is reduced and vanishes completely for $q_0 = 0.9$. At the same time, a strong reduction of the existing low-order rational surfaces in the q-profile can be observed. These two observations provide a key for understanding the partial sawtooth reconnection. For conventional tokamak scenarios a monotonically increasing q-profile is characteristic. Thus, q_0 *determines the number of the low-order rational surfaces for a particular q-profile. Stochastization requires the existence of several low-order resonant surfaces which can be excited by the overlapping (1,1), (2,2) and (3,3) resonances.* Without these resonant surfaces, the (2,2) and (3,3) perturbations only slightly modify the shape of the (1,1) mode and the system is not stochastic at all (figure 2c)! It is interesting to note that for the considered experimental plasma perturbations the critical value for avoiding stochastization is $q_0 = 0.90 \pm 0.05$ (smaller than unity!). At the same time, the (1,1) island structure itself is always not stochastic, which means that the mode survives the crash.

It is remarkable that also transition to chaotic stage can be observed in some sawtooth crashes. Such transition from non-chaotic to chaotic/stochastic behaviour is one of the fundamental question in nonlinear science. Intensive investigations of completely different mathematical and experimental chaotic systems demonstrate that there are three possible roads to chaos [10]: (i) period doubling, when the period is doubling many times during the transition (ii) intermittency, which is characterized by sudden changes from non-chaotic to chaotic behavior and back; (iii) quasiperiodicity which is characterized by appearing of two incommensurable frequencies. Each of these roads to chaos has a set of unique signatures which appears in the system independently from its nature. It could be a physical, biological or any other system, but the roads remain the same. Thus, a set of transition signatures is a universal invariant which could be used to verify the nature of the system and clarify the type of the transition to chaos. Analysis of the experimental SXR and ECE signals before a sawtooth crash shows clear indications of quasiperiodical transition to chaos as shown in figure 3.

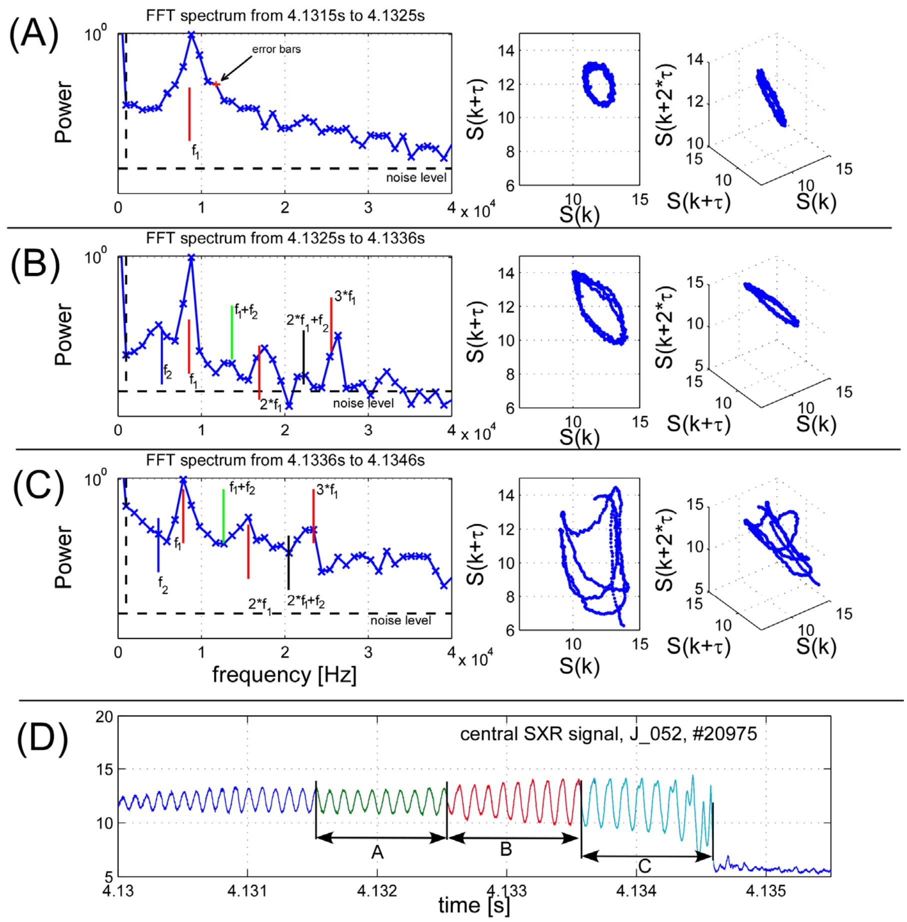

Fig. 3. Analysis of the sawtooth crash in ASDEX Upgrade (A-D). Normalized spectral amplitude and reconstruction of trajectory by means of delay coordinates (number of time points for the time delay: τ=600) are shown for single mode regime (A), slightly quasiperiodic regime which is close to the crash time (B), and strongly quasiperiodic regime just before the sawtooth crash (C). Smallest resolvable frequency and noise level are indicated by dashed lines. The SXR signal is shown in figure (D). Here f_1 is the frequency of (1,1) mode.

The dynamics of the instability before and during the sawtooth crash was studied by means of spectral analysis and reconstruction of phase trajectories using delay coordinates which are the standard techniques for analyzing stochastic systems and have been used for identification of the transition to chaos in different physical systems. It was demonstrated on the basis of the soft X-ray (linear integrated measurements) and electron cyclotron emission measurements (local measurements) that during the pre-crash phase the clear signatures of quasiperiodic transition to stochastic stage are present [11]:

- MHD oscillations with two incommensurable frequencies develop before the crash. Other peaks are linear combination of two primary frequencies.
- Reconstructed trajectories in 2D and 3D phase space demonstrate transition from single frequency behavior to strongly quasiperiodic regime. These plots were constructed from the same signal using the fixed time delay τ and are analogous to Poincare plots (2D) and torus phase structures (3D) in phase space. In these phase plots our transition to chaos should have the following steps: (single frequency) → (2D torus) → (3D chaos). Indeed, in the case of the single frequency f_1 we observe a planar periodic cycle which is typical signature of pure periodic behavior (Fig.3A). In the slightly quasiperiodic case open orbits are observed (Fig. 3B). In the last pre-crash phase strongly quasiperiodic behavior is seen which is characterized by a non planar 3D structure (Fig.3C) but the torus structure is not completely destroyed. (Completely chaotic stage is characterized by a cloud of the trajectory points in 2D plot and an attractor structure or cloud of points in 3D plot depending on system's nature.)
- Typical increase of broad band low frequency noise just before the crash is seen. This is also a typical feature of other quasiperiodic transition experiments [12].
- Moreover, consistent with the most energetically favorable transition from quasi-periodicity to chaos, frequency ratio between two modes is close to the golden mean ratio $G = f_2 / f_1 = \left(\sqrt{5}-1\right)/2 \approx 0.618$, which is the most irrational number [12].

All these results strongly suggest that stochastic model is one of the most probable explanations for the sawtooth crash.

Other interesting observation is the amount of correlation between the band powers of frequency bands [13] of the two incommensurable frequencies. Different sawteeth in different discharges gave roughly the same correlation value (about 0.6) within the error bars as shown in the table. We have observed definite time delays between the two bandpowers. The order of magnitude and direction of the time delay showed to be consistent. Possible explanations for relatively large correlation values are 1) a common origin for both frequency bands with different time pathes, 2) one of the oscillations affects on the other one and the time delay comes from that mechanism. The consistency of time delays and the constant value of the correlation also suggest that this is an independent indicator of the process. (It is important that the frequency of the (1,1) mode before the crash are different for these sawteeth.)

Shot	T_{begin} (s)	T_{end} (s)	Line	$Band_1$ (kHz)	$Band_2$ (kHz)	Correlation	τ_{peak} (ms)
20975	4.13	4.134	J_053	4-7	8-11	0.61±0.22	-0.3
22036	3.112	3.1185	J_054	4-8	10-13	0.61±0.17	-0.3
23068	2.5	2.518	J_053	7-13	15-20	0.52±0.24	-1.4
23068	2.71	2.727	J_053	6-11	13-20	0.52±0.15	-1.7

4. Conclusions

Our investigations demonstrate that stochastization in relatively small region of the plasma plays an important role in different MHD events. It was shown the favourable role of stochastization in the FIR-NTM regime, negative role of stochastization during the minor disruption and stochastic explanation of the sawtooth crash. All these events are important for plasma confinement. This means that stochastisity strongly influence on the plasma not only in case of the big stochastic regions and long time scales like in RFPs but also for short times and small region as it is typical for tokamaks.

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