Momentum Transport from Tearing Instability

S.C. Prager 1), A.F. Almagri 1), D.L. Brower 2), D. Craig 3), D.J. Den Hartog 1), W. Ding 2), F. Ebrahimi 1), A. Kuritsyn 1), M. Miller 1), V.V. Mirnov 1), J.S. Sarff 1), and the MST Team

1) University of Wisconsin, Madison, WI USA, and Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas
2) University of California, Los Angeles, CA USA
3) Wheaton College, Wheaton, IL USA
email: scprager@wisc.edu

Abstract. In the MST reversed field pinch the radial profile of the plasma rotation undergoes a rapid flattening during magnetic reconnection events. The reconnection events are repetitive (occurring as sawtooth crashes) and correspond to bursts of tearing instabilities. The radial transport of toroidal and poloidal momentum occurs faster than can be explained by collisions. We measure the change in momentum spectroscopically and we measure the fluctuation-induced stresses (Maxwell stresses by laser Faraday rotation and Reynolds stresses in the edge with probes). We find that the stresses are large, but nearly equal in magnitude and opposite in direction, such that their difference can account for the change in momentum. We also report theoretical investigations through quasilinear analytic calculation and nonlinear MHD computation that describe evaluated momentum transport from tearing modes, including their nonlinear coupling. The combined experimental and theoretical studies establish that tearing modes transport momentum, and that the transport is enhanced through nonlinear mode coupling.

1. Introduction

It is observed in numerous toroidal plasma configurations that the plasma rotates spontaneously, acquiring toroidal and/or poloidal momentum in the absence of external torques. In addition, it is observed that the radial transport of momentum is enhanced beyond that predicted to arise from Coulomb collisions. Momentum transport and torques in tokamaks have been studied for many years; however, the causes are not yet determined [1]. Electrostatic turbulence has been put forth as a possible mechanism.

In the reversed field pinch, the plasma also spontaneously rotates, and this momentum is observed to be transported radially at a rapid rate relative to classical expectation. In the MST reversed field pinch (RFP), this is most evident during sudden magnetic reconnection events—the crash phase of sawtooth oscillations.[2] During this brief time, the radial profile of the momentum becomes flatter. In this paper, we provide experimental and theoretical results that establish that tearing instabilities, and their associated magnetic fluctuations, produce momentum transport and are the likely explanation for this behavior in the RFP. This mechanism could play a role in tokamaks during periods of strong MHD activity.

The origin of the fairly large spontaneous rotation generation has not yet been identified in MST. Prior measurements in the far edge region of the reversed field pinch identify the spontaneous formation of plasma flow that is consistent with a turbulent Reynolds stress associated with electrostatic fluctuations.[3,4] Typically these measurements have been made in more quiescent periods that are not punctuated by large bursts in magnetic tearing reconnection.

Transport of momentum is determined through measurements of the changes in the rotation profile, reported in Section 2. The mechanism is established experimentally through measurement of Maxwell and Reynolds stresses arising from tearing instability, described in Section 3. Theoretical examination of momentum transport from tearing modes is described in Section 4, with conclusions in Section 5.

2. Measurement of momentum transport

Experiments are carried out in the MST reversed field pinch (a = 0.5 m, R = 1.5 m, current up to 0.5 MA) in Ohmically heated plasmas (no external momentum drive). MST exhibits sawtooth oscillations, during which the magnetic fluctuations surge and the current density profile flattens. The current density transport process has long been studied, and is associated with tearing instability (the "dynamo effect"). Sudden changes in plasma flow are also observed.[2] For example, the core toroidal flow decreases during sawtooth crashes, as shown in *FIG.1*.

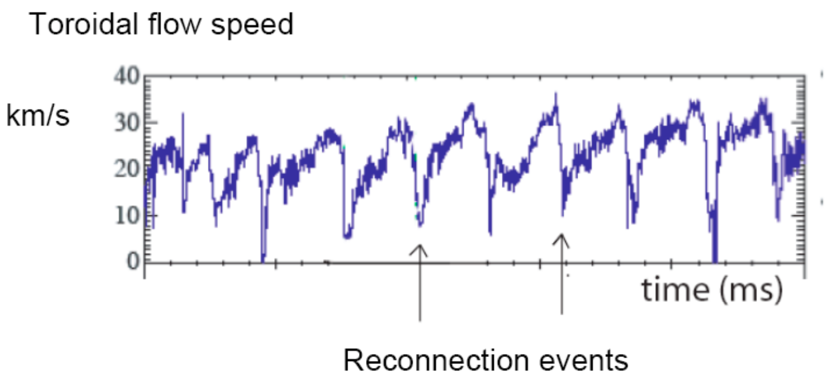

FIG.1. Toroidal flow speed in the center of the MST plasma, showing sudden change during reconnection events (sawtooth crashes).

This change in the flow corresponds primarily to a radial transport of momentum, rather than a net loss from the plasma. This is illustrated by comparing the core flow and the edge flow during the sawtooth crash, as shown in *FIG.2*. The flow at different radii becomes nearly equal during the crash, indicating flattening of the profile. The displayed flows are parallel to the magnetic field (toroidal in the core, poloidal in the edge), illustrating flattening of the parallel flow profile. The time scale for the flattening (~ 100 μs) is much faster than expected from collisional viscosity. Coincident with the

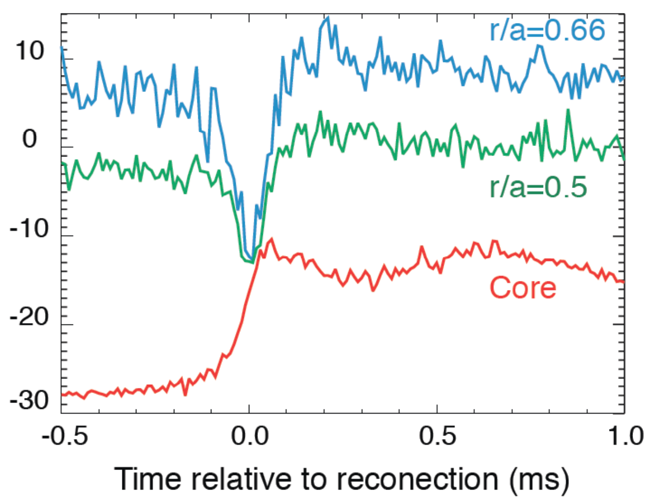

FIG.2. Plasma flow, parallel to the magnetic field, measured in the MST core and outer region through an individual sawtooth crash (reconnection event).

momentum transport is a surge of magnetic fluctuations (*FIG.3*), which have dominantly poloidal mode numbers m=0,1 and a range of toroidal mode numbers, n. This suggests causality between the momentum transport and magnetic fluctuations, which are known to be tearing instabilities.

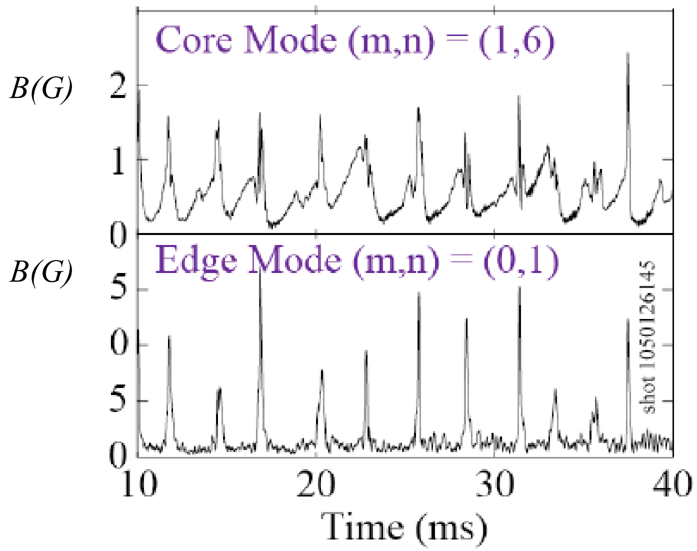

FIG.3. Magnetic fluctuation amplitudes of a core-resonant mode (m = 1, n = 6) and an edge resonant mode (m = 0, n =1), showing bursts at the sawtooth crashes

3. Measurement of fluctuation-induced stresses

Fluctuations can alter the plasma flow through the Maxwell and Reynolds stresses, as indicated from the momentum balance equation for the mean flow

$$\rho \frac{\partial \langle v \rangle_{\parallel}}{\partial t} = \langle \tilde{j} \times \tilde{B} \rangle_{\parallel} - \rho \langle (\tilde{v} \bullet \nabla) \tilde{v} \rangle_{\parallel} \qquad (1)$$

where $\langle\ \rangle$ denotes a magnetic surface average (or mean quantity), and tilde denotes fluctuations. (The parallel momentum balance is analyzed for its connection to the measurements in *FIG.2* and discussion below.) The first term on the right side is the fluctuation-induced Lorentz force (related to the Maxwell stress) and the second term contains the Reynolds stress. In the edge of MST (the outer 15% in minor radius) all three terms have been measured with probes (magnetic probes for magnetic field and current density, and Mach probes for flow). The measured three terms are shown in *FIG.4*. Both the Lorentz force and the Reynolds stress term are large. Surprisingly, they are each much larger than the inertial term (the left side of Eq. 1). On the scale of the plot, the nonzero change in the inertial term is not evident. Interestingly, the Lorentz force and Reynolds stress term are of comparable magnitude, but opposite in direction, and the difference between them is comparable in magnitude to the inertial term.

This result in the MST edge is consistent with measurements in the core. In the core, the flow and inertial term are measured with charge exchange recombination spectroscopy. The Lorentz force is measured with laser Faraday rotation, which is capable of measuring magnetic field and current density fluctuations. It is observed that the Lorentz force in the

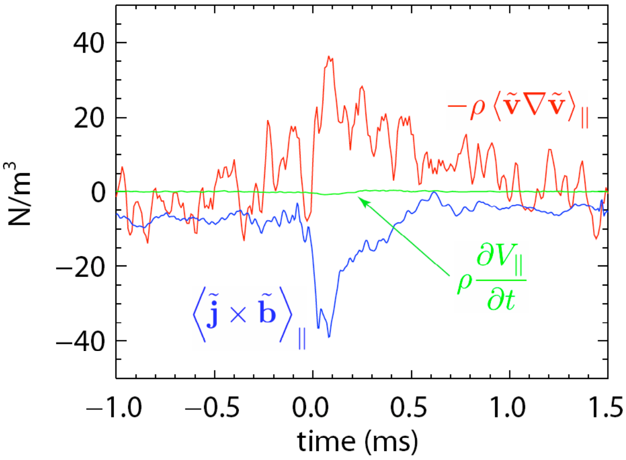

FIG.4. Measurements of three terms in Eq. 1, through a sawtooth crash, in the MST edge region.

core is much larger than the inertial term (the Reynolds stress has not yet been measured in the core).[5]

The strong momentum transport observed at the sawtooth crash depends on whether or not nonlinear coupling between core-resonant m=1 tearing modes and edge-resonant m=0 modes occurs. This is illustrated in *FIG.5*, which shows the change in the core rotation depending on the appearance of a large increase in the m=0 edge mode. Occasionally there are events in which only the m=1 mode increases in amplitude, but the m=0 amplitude remains small (the red traces in *FIG.5*). The momentum transport is observed large only when both m=1 and m=0 modes burst to large amplitude. The m=0 mode has been measured to be linearly stable during the sawtooth crash, implying that its burst in amplitude results from nonlinear coupling to the known unstable m=1 modes in the core.[6] Other magnetic relaxation processes, e.g., the dynamo effect, possess this same behavior, large and global at the sawtooth crash only when the nonlinear coupling between m=1 and m=0 modes is strong.

An obvious question is why the individual fluctuation-induced forces are so much larger than the inertial term. The reason might be related to the coupling of the momentum transport to the current density transport. The Lorentz force of Eq. 1 also enters as a fluctuation-induced

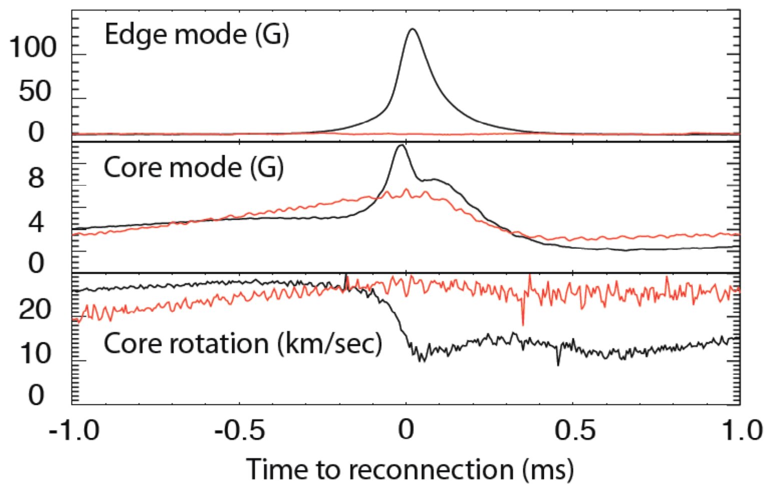

FIG.5. Magnetic field amplitudes of the dominant edge-resonant m=0, n=1 mode, core-resonant m=1, n=6 mode, and core plasma rotation for cases with (black traces) and without (red traces) a large increase in the m=0 mode amplitude.

Hall term in the mean-field parallel Ohm's law,

$$E_{\parallel} - \eta J_{\parallel} = -\langle \tilde{v} \times \tilde{B} \rangle_{\parallel} + \langle \tilde{j} \times \tilde{B} \rangle_{\parallel} / n_e e \ . \tag{2}$$

The first term on the r.h.s. of Eq. 2 is the MHD dynamo, while the second term is referred to as the Hall dynamo in the context of current profile relaxation (i.e, parallel current transport). The Hall dynamo is a two-fluid effect which has been measured large near major mode-resonant surfaces in MST.[5] Thus, a large Hall dynamo (that relaxes the current density profile) would also produce a strong effect on plasma momentum. If the plasma has a preferred flow profile (e.g., mandated by flow effects on tearing instability), then a strong, opposing Lorentz force would need to arise. Although the coupled momentum and current transport problem has not been solved from two-fluid theory, we have examined in detail the momentum transport from tearing modes in single-fluid MHD, described in the next section.

4. Theoretical and computational evaluation of stresses

To elucidate the physics of the forces (or torques) from tearing modes, we have performed three sets of calculations, of increasing completeness.[7] Each treats an RFP plasma with sub-Alfvenic flow. First, we have analytically examined forces arising from a single tearing mode in the linear regime. The force from both the Maxwell and Reynolds stresses, evaluated from quasilinear theory, are localized to the narrow reconnection layer.

Second, we have computed the forces for the full nonlinear evolution of a single tearing mode, In the linear regime, the computation is consistent with the quasilinear theory. The forces in the nonlinearly saturated state are shown in *FIG.6*, where the relative contributions of the two forces depends on the values chosen for the resistivity and viscosity. Both forces change sign about the mode-resonant surface, thereby transporting momentum outward in radius (while conserving the total momentum in the plasma). The transport rate is 1000 times faster than that due to classical viscosity.

Third, we have computed the complete case of multiple, nonlinearly coupled tearing modes. The computation, using the DEBS code, solves the full MHD equations. Multiple modes arise and follow an approximate sawtooth cycle, somewhat similar (though less sharp) to that which occurs in experiment. The presence of multiple modes leads to a global change in the flow profile, as shown in *FIG.7.* In addition, comparison to the single nonlinear mode reveals that nonlinear coupling strengthens the turbulent stresses. The effect of multiple, nonlinearly coupled tearing modes is not merely the superposition of independent, radially separated effects. Rather, the force arising from the stress of one mode (among many) is itself increased by the presence of other modes. For example, the phase between the current density and magnetic field of a specific mode is altered (from the case of one mode only) so as to increase the Maxwell stress. This effect is consistent with experiment. During a sawtooth crash, the phase between $\tilde{j}$ and $\tilde{B}$ (measured by Faraday rotation) for a core-resonant tearing mode changes to increase the Maxwell stress term.[5] Finally, to further test the effect of nonlinear coupling, we have performed a computation in which the $m = 0$ mode is computationally suppressed, much like the experimental case shown in *FIG.5*. This suppresses the dominant nonlinear coupling effects. In this case the plasma evolves to a quasisteady state without sawtooth oscillations, the forces and momentum transport are reduced greatly, and the flow profile does not flatten, and momentum transport is greatly reduced.

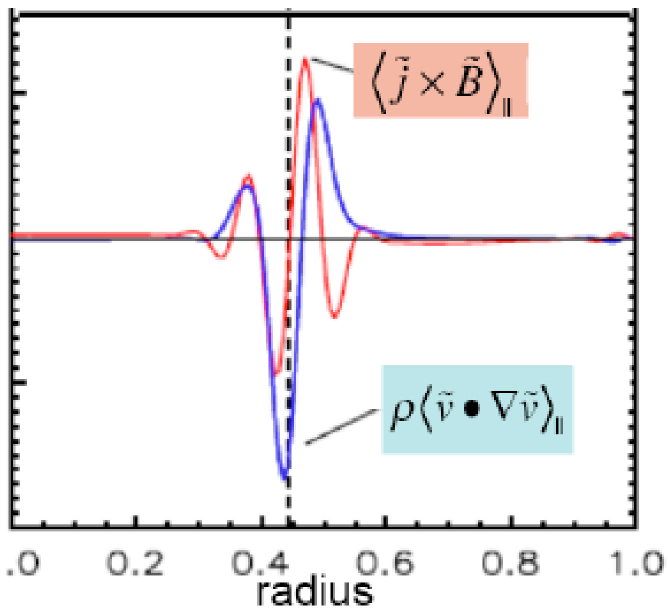

FIG.6. MHD computation of instability-induced forces from a single tearing mode, vs radius, in its saturated state. The vertical dashed line denotes the radial location of the mode-resonant surface

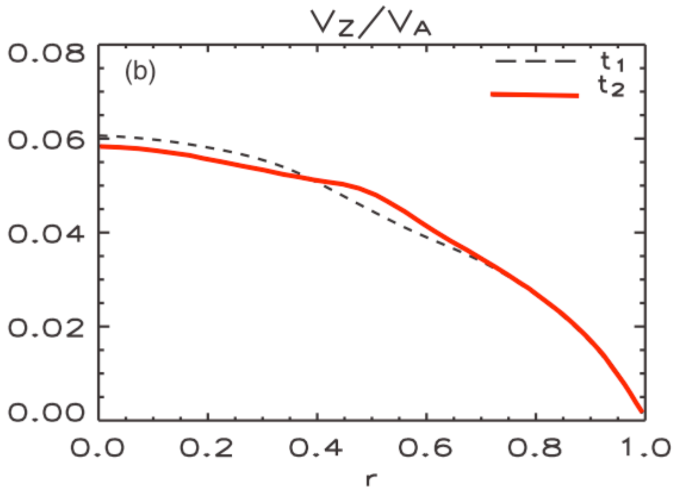

FIG.7. Radial profile of axial plasma flow before (dotted line) and after (solid line) as sawtooth crash, as predicted from MHD computation with multiple tearing modes.

5. Conclusions

Experimental measurements of momentum transport and fluctuation-induced forces, combined with theoretical and computational MHD evaluation of the forces, establish that tearing instabilities drive strong momentum transport. Moreover, the transport is enhanced by the nonlinear coupling of multiple tearing modes. However, two experimental results are yet not fully understood. First, the two fluctuation-induced forces (from Maxwell and Reynolds stresses) are each much larger than needed to explain momentum transport. The total force (the sum of the two large, but opposing forces) is of the right magnitude (although within error bars) to account for the transport. The reason for the large size of the individual forces, not yet known, is conjectured to arise from the coupling of the momentum transport problem to the two-fluid dynamo problem, within which the Lorentz force appears as a Hall dynamo effect. Second the theoretical MHD results do not quantitatively match experimental results, producing smaller forces and weaker flattening of the flow profiles. These issues might be resolved through two-fluid investigation of momentum transport for parameters (such as resistivity and viscosity) closer to that of the experiment.

These results in a fusion plasma have connections to momentum transport in astrophysical plasmas. For example, momentum transport occurs at rapid rates in accretion disks that surround compact objects such a black holes. A transport rate higher than the collisional value must occur in order to account for the observed accretion rate. We are investigating momentum transport by current-driven tearing instability as a complementary explanation to the standard model of transport from flow-driven instability (known as the magnetorotatonal instability).

References

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